# Initial Primary Teacher Education 

## Mathematics

## Module 2

of

## Year 1

# Initial Primary Teacher Education 

## Mathematics

## Module 2 <br> of <br> Year 1

## Prepared and published by

Malawi Institute of Education PO
Box 50
Domasi Malawi
email: miedirector@sdnp.org.mw
website: www.mie.edu.mw
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## Foreword

Education is the lifeblood of the nation. It is a prerequisite for individual, community and national development. Education prepares learners to play their roles effectively to promote and sustain a country's socio-economic development. Parents or guardians desire that their children develop into adults with sound minds and healthy bodies through the acquisition of appropriate knowledge, skills and desirable attitudes to enable them to live productive and happy lives.

Education should, therefore, help learners to develop high standards of conduct, attitudes, judgment and a sense of responsibility. Student teachers have to be well prepared in order to be able to take this responsibility of teaching children effectively.

The provision of quality education is based on many factors and a good quality of teachers is one of them. Teachers play a central role because they are the key source of knowledge, responsible for facilitating the learning process and act as role models for the learners.

The function of initial teacher education in Malawi is to prepare student teachers in their aspiration of becoming teachers of high quality. This is achieved by helping the student teachers to acquire the right knowledge, skills and competences to enable them to effectively teach children. In view of this, the Initial Primary Teacher Education curriculum has been reviewed to ensure that student teachers who graduate from this programme are well trained and prepared for their profession.

The process and implementation of this review has been guided by the Teacher Education Philosophy which states as follows:
'To produce a reflective, autonomous, lifelong learning teacher, able to display moral values and embrace learners' diversity.'

It is therefore hoped that Teacher Training Colleges will find this curriculum effective in helping the student teachers to build a solid foundation in their teaching profession.

## Executive Director

Malawi Institute of Education

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## Production team

Editing
Designer
Editor-in-chief

Jackson Yekha
Thabu Mwenelupembe-Phiri
Max J Iphani

## Writers

| Getrude Jumbe | - Blantyre Teachers' Training College |
| :--- | :--- |
| Eneya Phiri | - Blantyre Teachers' Training College |
| Paschal Kayange | - Karonga Teachers Training College |
| Gabriel Chamdimba | - Machinga Teachers Training College |
| Bruno Chikopa | - Machinga Teachers Training College |
| Adhija Nangoma | - Marryiam Girls Teachers Training College |

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## Introduction

The purpose of primary teacher education is to produce and continually develop competent and responsive teachers who effectively deliver quality education to all learners under prevailing conditions and demands in primary schools and promote their desire for life-long learning. IPTE endeavors to educate teachers in sufficient numbers, continually develop their professionalism so that they are able to effectively and efficiently deliver quality and relevant education to primary school learners.

## National goals for primary teacher education

The national goals of primary teacher education in Malawi are to produce teachers who are:

- academically well-grounded and professionally competent
- flexible and capable of adapting to the changing needs and environment of the Malawian society
- capable of adhering to and maintaining the ethics of the teaching profession imaginative in adapting, creating and utilising locally available resources suitable for the needs of their learners.


## Rationale

Mathematics education aims at developing student's critical awareness of mathematical concepts and their relationships and how these are used for solving practical problems in a social, environmental, cultural and economic context.

At an early stage, the learners will be able to count and carry out basic mathematical operations. At a later stage, the learners will be able to make inferences using manipulated data and to apply mathematics for solving practical problems in daily their life.

## Teacher education philosophy

The following has been the guiding principle during the design, development and implementation of the IPTE curriculum.

To produce a reflective, autonomous, lifelong learning teacher, able to display moral values and embrace learners' diversity.

## IPTE programme structure

The duration of the teacher education is two years. The general outlook of the two years is as follows:

| Year 1 | Term 2 | Term 3 | Tear 2 | Term 1 | Term 3 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Term 1 | Th | Ter | In college, <br> with special <br> emphasis on | In college, <br> with special <br> emphasis on |  |
| In college, <br> learning <br> subject <br> content with a <br> special focus <br> on methods <br> for lower <br> classes | In college, <br> learning <br> subject <br> content with <br> special focus <br> on methods <br> for upper <br> classes | practice schools, <br> practising <br> teaching mainly <br> in the lower <br> classes | Out win <br> teaching <br> practise schools, <br> practising <br> teaching mainly <br> in the upper <br> classes <br> inclusion and <br> further <br> practice on <br> teaching <br> methods | content, <br> policies and <br> frameworks |  |

## Unique features

The features of the reviewed curriculum are as follows:

- The curriculum design is based on reflective and practice principles.
- Early grade teaching methodologies are distinct.
- The delivery of the subject content follows the modular approach.
- Student teachers will be allowed to practise teaching both in the lower classes (Standards 1 to 4 ) as well as in upper classes (Standards 5-8).
- Cross cutting issues such as Assessment for Learning, Information Communication Technology, Inclusive Education and Critical Thinking are integrated.


## IPTE subject matrix

The new curriculum has adopted the reflective practitioner model of teacher education which connects reality and theory and integrates content and pedagogy in teaching and learning. In this structure, student-teachers will be in college for terms 1 and 2 of year 1 and be in primary schools for teaching practice in the third term of year 1 . Students will be back to college in terms 2 and 3 of year 2 to continue learning subject content, reflecting on their experiences during teaching practice and then wind up their studies.

This curriculum has adopted a modular design and contains eleven subjects. These are Chichewa, English, mathematics, education foundation studies, agriculture, social studies, life skills, science and technology, expressive arts, religious studies and human ecology. In this modular design, a set of units with related content forms a module in a subject. A module consists of 40 contact hours.

Although the programme is modular, four subjects: Chichewa, English, mathematics and education foundation studies will be offered throughout the two years while the remaining subjects will be spread across the two years.

## IPTE outcomes based curriculum

An outcomes-based curriculum is focused on students' achievement. To achieve the outcomes, the students are introduced to new knowledge in the context of their existing knowledge so that they can develop new understandings. Therefore, the process of learning is integral to the final product. These products are the outcomes, which student teachers achieve in terms of teaching competencies and must be clearly stated before they begin teaching. The achievements made at college however will only be seen to be truly beneficial when student teachers transfer the knowledge, skills and desirable attitude beyond college and view learning as a life-long process. This is considered essential to keep pace with the transition from college to practical classroom experiences.

There is need for student teachers to acquire knowledge, skills and desirable values and attitudes to enable them to implement the primary curriculum. To that end, student teachers should be fully conversant with foundation studies and other subjects taught in schools.

## Learning areas and core elements

A learning area is an organised body of the required knowledge, skills, values and desirable attitudes that serve as a foundation for future learning. Each learning area has a rationale from which core elements are derived. The IPTE curriculum comprises eleven learning areas namely agriculture, science and technology, mathematics, expressive arts, Chichewa, English, foundation studies, social studies, life skills, religious studies and human ecology. Each learning area has four or more core elements.

## Teacher education core element outcomes

Teacher education core element outcomes are descriptions of the competencies to be acquired by the student teacher for successful teaching.

## Progression of learning areas into subjects in the primary senior phase

Student teachers should know that during infant and junior phases, teaching and learning will centre on learning areas instead of isolated subjects. However, the curriculum will revert to subjects in the senior phase in line with the secondary school curriculum. For this reason, student teachers will study both learning areas and subjects.

## IPTE assessment procedures

In Outcomes-Based Education (OBE), assessment is a significant part of the teaching and learning process. The main purpose of assessment is to facilitate learning by
constant monitoring of the progress of individual learners. The process is on-going and it uses clearly defined criteria with a variety of tools, methods and techniques in different situations and contexts. This helps to gather valid and reliable information on the learners' achievement of outcomes.

Assessment in primary teacher education in Malawi comprises two major components: continuous and summative assessment. Both modes involve assessment tasks that measure the student teachers' achievement of knowledge, skills, values and attitudes. These tasks include oral presentations, practical tasks, reports, research, tests and examinations.

In the reviewed curriculum, the weighting of continuous assessment in the final grade will be $60 \%$ continuous assessment and $40 \%$ summative assessment.

The continuous assessment will comprise:

- two assignments based on each module
- end of module examinations excluding terms 2 and 3 of year 2
- teaching practice grades
- school experience journal grade

While the summative assessment will comprise:

- moderated grade from teaching practice in term 1 of year 2
- national examinations to be administered in term 3 of year 2 based on the modules of terms 2 and 3 of year 2

Summary of topics for the term and time allocation

| Term 1 |  |  |
| :---: | :---: | :---: |
| Topic | Allocated time in hours | Core element |
| Teaching processing of data | 4 |  |
| Teaching measures of central tendency | 2 | Data handling |
| Teaching HCF and LCM | 4 |  |
| Teaching fractions | 12 |  |
| Teaching decimals | 1 |  |
| Teaching approximation and estimation | 2 | Number concepts and operations |
| Teaching rate, ratio and proportion | 10 |  |
| Teaching capacity and volume | 5 | Measurement |

## TOPIC 1 Teaching of processing data

Time 4 hours

## Introduction

Processing data is one of the topics that are taught in Mathematics at primary and secondary school level in Malawi. It involves gathering information from real life situations, organizing and interpreting.
Processing data aims at visualizing the collected information for easy understanding by the user. The topic provides knowledge and survival skills such as decision making and problem solving. It will also help learners in later years to use the competencies acquired in pursuing different careers like research, accounting and economics. In the topic, you will analyse how learners develop concepts of processing data and apply appropriate teaching, learning and assessment methodologies in teaching of processing data

## Success criteria

By the end of this topic, you must be able to:

- analyse how learners develop concepts of processing data
- apply appropriate methodologies in teaching and learning of processing data
- apply appropriate assessment methodologies in the teaching and learning of processing data


## Background information

Processing data involves gathering information, organizing and interpreting processed data. Data is organized and interpreted through tables and graphs. Types of graphs covered in primary school curriculum are physical, picture, bar, line and pie charts. Physical graph uses real objects. Learners' queuing at the assembly forms part of the physical graph. Pictograph is sometimes referred to as picture graph. Real objects are represented by drawing. Bar graph is a representation of data on a Cartesian plane which uses rectangular bars with lengths proportional to the values of the items. The bars are either horizontal or vertical. Line graph is drawn by joining coordinates or pair of corresponding values of the given data in both axes. This type of graph shows trends or general patterns in things for example, antenatal card, and temperature. A pie chart is a graph where data is represented in sectors. The angle of the sector is proportional to the amount of data. A pie chart is drawn by converting the data given into degrees proportionally.
Using technology such as calculators and computer simplifies the task of
processing data. Most learners find it challenging working with data processing especially in drawing graphs to a given scale and interpreting them. However, use appropriate teaching, learning and assessment methodologies In the teaching and learning process, knowledge development progresses from what learners already know to what is new. Understanding learners' prior knowledge data processing will help teachers to develop appropriate strategies and resources to use. This task focuses on establishing learners' prior-knowledge on processing data. It has also focused on the importance of these three concepts.

## Activity 1 Establishing learners' prior knowledge and importance of processing data.

1. Consider the following activity.
a. Explore activities that can be used to elicit learners' prior knowledge on processing data.
b. Discuss is the importance of data processing and its application in life.
c. Examine primary school curriculum materials to determine expected achievements on data processing at each level
d. Share your work with the class.

Exploring appropriate teaching and learning methodologies for teaching data processing
may minimize their challenges.

## Developing the concepts measures of central tendency

Teaching data processing to learners of different classes and ability levels requires use of appropriate teaching and learning methodologies.
In this task, you will be introduced to strategies for teaching concepts of data processing such gathering information, organizing and interpreting data.

## Activity 1 Exploring meaning of data,

 sources of data, and ways of gathering of dataa. Collect different sets of data in the classroom. For example, shoes sizes and age.
b. Apart from the source you used to get the data, explore other sources of data.
c. Describe the technique you used to collect data.
d. Explore other techniques that can be used to collect data
e. Generate the meaning of data

## Activity 2 exploring ways of organizing data

1. Research on strategies of organizing data.
2. Using any strategy identified in 1, explain how you would organize data. .
3. Discuss activities that you would use to draw physical graphs, picture
graphs, bar graph, line graph and pie chart.
4. Report to the class.


Assessment complements the teaching and learning process. In this task, you will explore appropriate assessment processing. This will help you to get necessary feedback from your learners and eventually use appropriate methodologies to support them.

## Activity 1 Discussing ways of

 assessing learners on data processing.a. Prepare a lesson plan on how you can teach one of the graphs.
b. Micro-teach the lesson.
c. Explore misconceptions and errors learners may have on data processing.
d. Explain the sources and how to correct the misconceptions and errors.

## Activity 2 Interpreting data processing

1. Consider the graph below:

## Exploring appropriate

assessment methodologies in the teaching and learning of processing data
methodologies in the teaching and learning of the data

i. How many students prefer yellow juice?
ii. How many more students does an orange juice have over red juice?
iii. How many students were there altogether?

## Summary

Data is any set of collected information. The data is collected from different sources like football games, weather stations, hospitals and classrooms. The data collected is then processed into information to make meaning to the reader. Data can be presented in tables and graphs for easy storage, reading
and interpretation. The commonly used tables are frequency distribution table. There are five types of graphs that are taught in primary school. These are physical, picture, bar, line and pie chart.

Draw a bar graph and pie chart using the information in the table.
2. Using the same table, practice drawing bar graphs using Microsoft Excel software package.
3. The following are the weights of babies born at Balaka district hospital in the month of May, 2017: $2.1 \mathrm{kgs}, 1.9 \mathrm{kgs}, 3.2 \mathrm{kgs}, 1.8 \mathrm{kgs}, 2.0 \mathrm{kgs}$, $2.7 \mathrm{kgs}, 1.9 \mathrm{kgs}, 3.1 \mathrm{kgs}, 1.5 \mathrm{kgs}, 3.2 \mathrm{kgs}$, $1.7 \mathrm{kgs}, 1.9 \mathrm{kgs}, 3.1 \mathrm{kgs}, 1.9 \mathrm{kgs}, 1.6 \mathrm{kgs}$ and 1.9 kgs
i. Present the data in frequencies distribution table.
ii. Describe the rules used while generating the tallies and the frequencies.
iii. Develop an activity that you can use to teach learners organizing data.
iv. Present the activity to the class for discussion and evaluation.
4. Below is a line graph. Use it to answer questions that follow

## Reflection and assessment

1. The table below present data of number of shoes manufactured at Mbayani Shoe Company.

| Type of <br> shoe | Nkuna | Square | Canavas | Gogoda |
| :--- | :--- | :--- | :--- | :--- |
| Number <br> of shoes | 15 | 20 | 30 | 45 |


i. How many bags does Chedwa maize variety produce per hectare?
ii. Which variety produced more bags?
5. The following table shows the number of trees at three agriculture research stations

| Research <br> station | Number of <br> trees |
| :--- | :--- |
| Namikasa | 144 |
| Nyangu | 60 |
| Chiziwiziwi | 96 |

Draw a pictograph. Use 1 picture to represent 12 trees
6. The picture graph shows number of people in some townships of Manama District. Use it to answer questions that follow
7.

Township Number of people

Mtukulamwendo $\stackrel{+(+9+9}{+9+9}+$


If $\stackrel{\ominus}{+}_{\lambda}$ represent 14 people,
i. How many people are in

Mtuwambuzi Township?
ii. How many more people are in Majiga than in Mphandula?
iii. Find total number of people in the Townships?

Table 2 shows frequency distribution table of grades scored by standard 7 learners in a Mathematics test.
Table 2

| Marks (\%) | Tally | Frequency |
| :---: | :---: | :---: |
| 0-9 | II | 2 |
| 10-19 | IIII | 4 |
| 20-29 | III | 3 |
| 30-39 | II\#\# | 10 |
| 40-49 | III IIII | 9 |
| 50-59 | \#\#\# IIIIIIH- | 15 |
| 60-69 | IIII | 4 |
| 70-79 | II | 2 |
| 80-89 | III | 3 |
| 90-100 | 1 | 1 |

a) How many learners wrote the test?
b) How many learners passed the test if the pass mark was $50 \%$ ?

## Glossary

Bar graph a bar chart or bar graph is a chart or graph that presents grouped data with rectangular bars with lengths proportional to the values that they represent.
Data any set of collected information
Frequency the number of times a data value occurs
Tally

Axes a quick way of keeping track of numbers in groups of five dimensions of a Cartesian plane

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## TOPIC 2

## Teaching of measures of central tendency

Time 2 hours

## Introduction

Measure of central tendency is a branch of Mathematics taught in both primary and secondary school in Malawi. It involves three descriptive statistical concepts which are mean, mode and median.

Measures of central tendency provide visual image of the given data. As such, they provide a basis for comparing. An understanding of measures of central tendency assists learners to build strong foundation from which to build their future career in fields like statistics, demography, economics and even education.

This unit focuses on analyzing how learners develop the concepts of mean, mode and median. You will also explore appropriate teaching, learning and assessment methodologies for teaching mean, mode and median.

## Success criteria

By the end of this topic, you must be able to:

- analyze how learners develop the concepts of measures of central tendency
- apply appropriate methodologies in teaching and learning the
concept of measures of central tendency
- use appropriate assessment methodologies in the teaching and learning of measures of central tendency


## Background information

Student teachers have some knowledge on the three measures of central tendency. These are mean, mode and median. Most of the student teachers are knowledgeable on how to calculate each of these measures. Mean is calculated by adding all data sets and dividing the sum by the total number items. Mode is the value that occurs or appears most in a set of data. Finally, median is defined as the middle number in a set of data which is in either descending or ascending order. In some cases, it is possible to have no mode in a set of data, and sometimes a list can have one mode or more than one mode.

These concepts are well understood by learners when teachers use appropriate teaching, learning and assessment methodologies that involve learners in the lesson. Teachers need to use variety of activities and examples to assist learners understand the concepts and minimize their confusion.

## Developing the concepts of measures of central tendency

In the teaching and learning process, knowledge development progresses from what learners already know to what they don't know. Understanding learners' prior knowledge in mean, mode and median will help teachers to develop strategies and resources to use. This task focuses on establishing learners' prior-knowledge on mean, mode and median. It has also focused on the importance of these three concepts.

## Activity 1 Establishing learners' prior knowledge on measures of central tendency

1 Think of ways how learners' prior knowledge on mean, mode and median would be established.

2 Discuss the importance of mean, mode and median in life.

3 Explore reasons why mean, mode and median are called measures of central tendency.
4 Share your work with the class.
5 Examine the primary school curriculum materials and determine learners expectations on measures of central tendency at each level

## Exploring appropriate teaching, learning and assessment strategies for teaching mean, mode and median

Teaching mean, mode and median to different classes and levels of learners
requires use of appropriate teaching, learning and assessment methodologies. In this task, you will be introduced to strategies for teaching mean, mode and media.

Mean, which is also called, average is the mostly used measure of central tendency. Mean is important because it brings equality among scores in a set of data.

## Activity 1 Discussing strategies for teaching the concept of mean, mode and median

1 Consider the following data;
a. $8,7,5,8,4,8,9,6,8,7$, 4,3,1,4,2,2,4,3,1,2.
b. Explain how can teach mean, mode and median using the data.
c. Think of approaches you would use to collect data.
d. Formulate as many questions as you can on mean, mode and median.

```
TIP
The three concepts should be taught
```


## Summary

Measure of central tendency involves calculating mean, mode and median. Mean is the value found by dividing sum of the total data by the number of items in a set. Mode is the number that occurs most often in a set of data.
Median is the middle number in a set of data when it is written in ascending or
descending order. Teaching of mean, mode and median requires use of appropriate teaching, learning and assessment methodologies.

## Reflection and Assessment

1 a. Observe the colour of clothes learners are wearing in your class and record the data by colour and number of learners.
b. What approach was used to collect data in 1 a.
c. What are other ways of collecting data
2 From the data collected, calculate the most liked colour.
3 An average age for 15 learners in standard 6 class at Domasi Primary school is 12 years.
a. Calculate the number of learners in class.
b. Generate a set of data for the 15 learners whose average is 12 .
4 Explore ways errors learners would make in mean, mode and median
5 In a football team, 7 players had an averaged mass 55 kg and the average mass of the remaining 4 players was 61 kg . What was the average mark for the 11 players?
6 If the average of $3,6,4,10,13$ and $x$ is 6 , calculate the value of $x$.

## Glossary

Mean is the sum of data divided by the number of items in the set of data.
Median it is the middle number in a set of data that is written in order.
Mode is the score which is appearing most in a set of data.

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Time 4 hours

## Introduction

HCF and LCM is one of the topics in primary mathematics. Knowledge of HCF and LCM will help learners to understand and be competent in concepts of fraction.

In this unit, you will explore different concepts of HCF and LCM and appropriate methodologies that are used when teaching and assessing HCF and LCM in an inclusive setting.

## Success criteria

By the end of the topic, you must be able to:

- analyse how learners develop the concept of HCF and LCM
- use appropriate methodologies when teaching and learning HCF and LCM
- use appropriate assessment methodologies in the teaching and learning of HCF and LCM


## Background information

You learnt the concept of HCF and LCM in both primary and secondary school Mathematics. The concept of HCF involves finding the highest or greatest number (factor) that divides into a set of given numbers without leaving a remainder. It also involve modeling
factors; finding HCF using different strategies, such as factor method, prime factors, tree diagram and division methods; and working out word problems.
The concept of LCM involves finding the lowest or smallest number (multiple) that is divisible by a set of given numbers without giving a remainder. It also involves modeling multiples, find LCM using different strategies and working out word problems.

Teaching HCF and LCM requires use of relevant resources, appropriate methodologies and assessment. Learners often confuse in differentiating the two because they expect to find the bigger number in HCF and smaller number in LCM. Clear knowledge of HCF and LCM will assist learners to understand fractions.

## Ways of developing the concepts of HCF and LCM

There are different strategies how learners develop the concept of HCF and LCM. This depends on how they assimilate what they are taught with real life situations.

## Activity 1 Analysing ways how Learners develop the concepts of HCF

1 Using your primary experiences, discuss how you developed the ideas of HCF.
2 Explain the importance of HCF in real life situation.

## Activity 2 Analysing ways how Learners develop the concepts of LCM

1 Using your primary experiences, discuss how you developed the ideas of LCM
2 Explain the importance of LCM in real life situation.

## Appropriate teaching, learning and assessment methodologies when teaching of HCF

Teaching of HCF involves several teaching, learning and assessment methodologies. The following activities will help you explore and reflect on these methodologies and misconceptions.

## Activity 1 Analysing activities in teaching HCF

1 Using teachers guides and learners books from primary school curriculum, analyse the materials to find out the activities learners are involved in HCF

2 Summarise the findings by completing the table below

| Class | Activities | Other topics <br> that involve <br> HCF |
| :--- | :--- | :--- |


|  |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |

3 Present the finding to the class

## Activity 2 Exploring strategies for teaching and learning HCF

1 Identify strategies for teaching HCF
2 Analyse advantages and disadvantages of each strategy
3 Using the identified strategies, prepare a lesson plan.
4 Present your lesson.
5 Analyse advantages and disadvantages of each strategy.

## Appropriate teaching, learning and assessment methodologies when teaching of LCM

Teaching of LCM involves several teaching, learning and assessment methodologies. The following activities will help you explore and reflect on these methodologies and misconceptions.

## Activity 1 Analysing activities in teaching LCM

1 Using teachers guides and learners books from primary school curriculum, analyse the materials to find out the activities learners are involved in LCM

2 Summarise the findings by completing the table below

| Class | Activities | Other topics that <br> involve LCM |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

3 Present the finding to the class for evaluation.

Activity 2 Exploring strategies for teaching, learning and assessing LCM
1 Identify strategies for teaching LCM
2 Using the identified strategies, prepare a lesson plan.
3 Present your lesson.
4 analyse advantages and disadvantages of each strategy.

## Activity 3 Analyzing learners' misconceptions, and errors on HCF and LCM

1 Analyse learner written work on HCF and LCM to identify misconceptions and errors
2 Find possible causes for the misconceptions and errors.
3 Reflect on your own teaching and find solutions to overcome the challenges.
4 Report the results to the class


HCF involves finding the greatest factor that can divide into a given set of numbers without leaving any remainder. LCM involves finding the lowest common multiple that that is exactly divisible by a set of a given numbers. Learners can calculate HCF and LCM using prime factor, factor and division methods. HCF and LCM often confuse in differentiating the two because they expect to find the bigger number in HCF and smaller number in LCM.

## Reflection and assessment

1 Find the HCF of the following set of numbers
a) 48,108 and 140
b) 777 and 1147
c) $\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$ and $\frac{9}{10}$

2 Calculate the LCM of the following
a) 3 and 9 (by modeling)
b) $2,4,6,8$ and 10

3 Mayamiko has two pieces of cloth. One piece is 72 cm wide and the other piece is 28 cm wide. She wants to cut both pieces into strips of equal width that are as wide as possible. What will be the width of the strips?
4 Two baskets contain 195 and 250 bananas respectively, which are distributed in equal number among children. Find the largest number of bananas that can be given so that 3 bananas are left over from the first basket and 2 from the second.
5 The following are some of the concepts that are covered in primary
school curriculum: factors, multiples, finding HCF of numbers, common multiples and finding LCM of numbers
i. arrange these concepts in correct teaching order
ii. identify relevant resources for teaching the concept of common multiples
iii. choose one concept and present a lesson in the classroom
\(\left.\left.\left.$$
\begin{array}{ll}\text { Glossary } & \\
\text { Factor } & \begin{array}{l}\text { a number which } \\
\text { divides into another } \\
\text { number without } \\
\text { leaving a remainder. }\end{array} \\
\text { Common factor a factor that is common } \\
\text { or appearing to a set of } \\
\text { given numbers } \\
\text { a whole number that } \\
\text { has two factors, one } \\
\text { and it self }\end{array}
$$\right\} $$
\begin{array}{l}\text { set of factors of a } \\
\text { number that are prime } \\
\text { numbers } \\
\text { Prime number factors }\end{array}
$$\right\} \begin{array}{l}a highest or greatest <br>
factor that is common <br>
to a set of given <br>

numbers\end{array}\right\}\)| numbers that are |
| :--- |
| exactly divisible by |
| given set of numbers |

Common multiple a number divisible by a set of numbers without leaving a remainder
LCM the lowest or smallest number divisible by a set of numbers without leaving a remainder

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## Teaching fractions

Time 12 hours

## Introduction

Fractions are widely applied in Mathematical concepts in primary school curriculum. Learners interact with fractions in their everyday life like sharing of things. Knowledge and competency in the concept of fraction will greatly assist them to make sound and informed decisions as well as understand other topics in the curriculum.

In the unit, you will analyse how learners develop concepts of fraction and exposed to appropriate teaching, learning and assessment methodologies of this concept. A good understanding of content and pedagogy will enable you help learners appropriately.

## Success criteria

By the end of the topic, you must be able to:

- analyse how learners develop concepts of fraction.
- apply appropriate methodologies in teaching and learning of the concept of fractions
- apply appropriate assessment methodologies in the teaching and learning of fractions


## Background information

Fraction is a part of a whole or part of a set of objects. Fractions are represented by two digits, one on top and the other at the bottom of a line called fractional bar. The top number is the numerator which shows how many parts are under consideration while the bottom number is the denominator which shows how many equal parts has the whole has been divided.


Primary school curriculum covers three types of fractions which are proper, improper and mixed.

This topic focuses on modeling fractions, ordering fractions, equivalent fractions, basic operations on fractions, generating word problems, learners' misconceptions and errors. Teaching fractions requires appropriate teaching, learning and assessment resources and methodologies. Most often, learners find working with fractions difficult despite using the idea in many real life situations. They also have problems when transiting from whole numbers to fractions especially on basic operations. Appropriate knowledge gained from fractions will form a foundation for
learners to understand other related topics like decimals; percentages; and rate, ratio and proportion in their primary curriculum.

## Ways of developing the concepts of fractions

Learners have some knowledge of fractions before joining formal education developed from real life situations. However, teachers need to use different strategies to help learners develop further the concept of fraction.

## Activity 1 Eliciting learners' prior knowledge on fractions.

1 Think of situations in life from which learners would develop the idea of fraction.

2 Explain how each situation is linked to fraction.

## Activity 2 Discussing importance of fractions

1 Consider the following fields and discuss how fractions are used:
a. medicine
b. food preparation

2 Think of other importance of fractions in everyday life.

3 Share with the class.


## Appropriate methodologies in teaching and learning of the concept of fractions

Teaching fractions to different classes and levels of learners requires use of appropriate teaching and learning methodologies. Many teachers approach fractions differently depending on their content and pedagogical knowledge. Teachers need to understand the instructional materials and skills expected at each level in the primary school curriculum. In this task, you will explore activities and appropriate teaching and learning methodologies for teaching the concept of fraction.

## Activity 1 Exploring activities in the teaching of fractions

1 Using teachers guides and learners books from primary school curriculum, analyse the materials to familiarise with the skills learners develop in different classes
2 Summarise the findings by completing the table below

| Class | Skills | Other topics that <br> involve fractions |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |

3 Present the finding to the class.

## Activity 2 Introducing fractions

1 Think of resources that you would use to introduce the concept of fraction.
2 Explain the strategy how you would the resources in introducing the concept.

3 Develop activities you would use to teach modeling, naming and writing of fractions eg. $\frac{1}{2}, \frac{2}{5}, \frac{3}{4}$.
4 Share with the class.

## Activity 3 Discussing equivalent

 fractions1 Consider the following fractions; $\frac{2}{3}$ and $\frac{10}{15}$.

2 Explore appropriate methodologies that you would use to show learners that the two fractions have the same value.

3 Explain how you would teach the concept of equivalent fractions using different resources.

## Activity 4 Ordering fractions

1 Model the following fractions using different resources. e.g. strips of rectangular paper of the same size. $\frac{1}{4}, \frac{1}{6}, \frac{1}{2}$ and $\frac{2}{3}$.
2 Compare the sizes of the fractions and order them in ascending.
3 Explore other strategies that can be used in ordering the fractions
4 Present your work to the class.

## Activity 5 Discussing types of fractions

1 Identify relevant teaching and learning resources that you can use when teaching each of the following types of fractions: proper, improper and mixed numbers.

2 Prepare a lesson plan and use it to teach each type of fraction using the resources identified.

3 Explore different strategies that would be used to differentiate each type of fraction from the other.
4 Present the work to the class.

## Activity 6 Discussing basic

 operations on addition and subtraction of fractions1 Consider the following question: $\frac{2}{5}+\frac{1}{5}$
2 Explore the strategies for teaching the question using different resources.
3 Explain how each strategy would be used to show that $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$.
4 Research and come up with different strategies and resources you would use when teaching:
i addition of proper fractions with different denominators
ii addition of mixed numbers with the same denominator. iii addition of mixed numbers with different denominators eg $2 \frac{2}{9}+5 \frac{1}{4}$
iv subtraction of proper fractions with the same denominator
v subtraction of proper fractions with different denominators vi subtraction of mixed numbers with the same denominator.
vii subtraction of mixed numbers with different denominators eg $5 \frac{1}{4}-2 \frac{2}{3}$
5 Share your work with the class.

6 Develop a lesson plan on one concept to practice the strategies

Activity 7 Discussing basic operations on multiplication and division of fractions.
1 Consider the following question: $\frac{2}{3} \times \frac{4}{5}$
2 Explore the strategies for teaching the question using different resources.

3 Explain how each strategy would be used to show that $\frac{2}{3} \times \frac{4}{5}=\frac{8}{15}$
4 Research on different strategies and resources you would use when teaching;
i Multiplication of a whole number by a fraction
ii Multiplication of mixed numbers. e.g. $2 \frac{3}{4} \times 2 \frac{1}{3}$.
iii Division of a proper fraction by a whole number
iv Division of whole number by a proper fraction
v Division of proper fraction by proper fraction
vi Division of mixed numbers .e.g. $2 \frac{3}{4} \div 2 \frac{1}{3}$.
5 Share your work with the class
6 Develop a lesson plan on one concept to practice the strategies

## Activity 8 Generating word problems involving fractions

1 Research on word problems involving fractions from your library and Primary school Mathematics textbooks.

2 Formulate your own word problems on fractions.
3 Discuss different strategies that would be used to solve them.
4 Share your work with the class.


## Appropriate assessment methodologies in the teaching and learning of fractions

Assessment is an integral component in the teaching and learning process. In this task, you will explore appropriate assessment methodologies in the teaching and learning of the concept of fraction. This will help you to get necessary feedback from your learners and eventually use appropriate methodologies to support them.

## Activity 1 Analysing learners' misconceptions, errors and strategies when solving fractions

1 Using digital camera to capture learners work on fractions from standard 3 to 8 from the
demonstration school or any nearby primary school.
2 Analyse and identify misconceptions and errors learners have on fractions .e.g. on writing fraction, problem solving strategies, in modeling and naming, etc.
3 Discuss the sources and causes of such misconceptions and errors.
4 Explore different ways how learners would be assisted.
5 Share with the class.

## Activity 2 Discussing ways of assessing learners on fractions

1 Explore other ways of assessing learners on fractions apart from written work.
2 Explain how each would be used.
3 Share with the class.
4 Prepare a lesson plan on any concept in fractions and micro-teach it.
5 Develop a checklist that you would use to assess learners' competency.
6 Evaluate the how the checklist has been used.


## Summary

A fraction is defined as part of the same whole. They are three types of fractions namely; proper fractions, improper fractions and mixed number/fraction. Improper fractions are fractions whose numerators are equal to or greater than their denominators. Equivalent fractions have the same value though the numerator and denominator and different. Teaching learners basic operation on fractions in the early years requires concretising the ideas through modeling. Solving problems involving two or more basic operations requires the use of the concept of BODMAS. Teachers need to use variety of teaching, learning and assessment methodologies when teaching fractions to cater for the needs of all learners.

## Reflection and assessment

1 Define a fraction.
2 Explain how you can teach the problem $\frac{2}{5} \div \frac{1}{3}$ by modelling.
3 Simplify $\frac{7}{9}-\frac{3}{5}+\frac{1}{2} \times \frac{2}{3}$
4 Simplify $\left(\frac{7}{16}+\frac{5}{8}\right) \times \frac{1}{5}$ of $\frac{2}{13}$
5 By how much is the quotient of $3 \frac{1}{4}$ and $2 \frac{4}{7}$ less than the sum of $5 \frac{2}{3}$ and $9 \frac{2}{3}$

6 Explore suitable resources for teaching the concept of fractions.
7 Write any two equivalent fractions of the following
i. $\frac{1}{3}$ ii $\frac{2}{5}$

8 Write any two problems learners may face when learning equivalent fractions.

## Glossary

| Fraction | part of the same whole |
| :--- | :--- |
| Numerator | the top number of a <br> fraction |
| Denominator | the bottom number of a |

Proper Fraction a fraction whose numerator is smaller than its denominator
Fractional bar a line that separates the numerator from the denominator
Improper fraction fractions whose numerators are equal to or greater than their denominators.
Mixed number/Mixed fraction a numeral composed of a whole number and a proper fraction
Equivalent fraction fractions which have the same value but different numerators and denominators
Unit fraction a fraction whose numerator is one
Modeling fraction using the objects to show the meaning of fraction

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## Reference

## TOPIC 5

## Teaching of decimals

Time: 1 hour

## Introduction

The Malawi primary school curriculum seeks to develop learners' critical calculational skills among many other skills. It is only learners with sound calculational skills who shall become reliable citizens. Developing learners' understanding in decimals is one way of producing reliable citizens. Learners' development in decimals is dependent on teachers' competence to handle such a topic. Teachers should have the skills that shall enable learners in primary schools be able to work with decimals. Teachers should therefore have a broader base of knowledge on how to confidently teach decimals. Decimals are so important in mathematics. They are not only important in a classroom setting but also in our every day's life. Decimals are used in a number of situations, for example in giving accuracies. They are further used when giving separations between units of quantities in a number for example in money (i.e. when separating the kwachas from the tambalas). Decimals form a very useful background to advanced college mathematics in students. Decimals are linked to higher level computations in mathematics.

## Success Criteria

By the end of this topic, you must be able to:
i. analyze how learners develop the concept of decimals.
ii. use appropriate methodologies in teaching and learning the concept of decimals.
iii. use appropriate assessment methodologies in the teaching and learning of decimals.

## Background information

The concept of decimals is totally hinged on the concept of fractions because decimals are an extension of fractions. Learners who have developed a strong foundation on fractions would not have challenges with decimals. Decimals are inseparable from the fractions. Therefore, their interdependence is vital in learners' understanding of mathematics in primary school and in later years. In Malawi primary school curriculum, decimals are taught from Standards 5 to 8. These standards have different scope on the coverage of decimals. Though the learners in these standards are learning mathematics in semi-abstract way, it is important to teach the learners with relevant teaching and learning resources
like spike abacus and place value charts. Modelling of decimals is equally an important element in the teaching of decimals. Decimals are classified as terminating or recurring. These classes are based on the behavior of digits in the quotients.
You must develop the skills of reading, writing and interpreting decimals before carrying out the basic operations. On basic operations on decimals, it is important to use different procedures in carrying out the operations. Use procedures that will allow learners to acquire basic knowledge on decimals through understanding and not through rote learning.
As learners are developing their understandings in decimals they may have a number of misconceptions. For example, learners may think that the place values for tenths is the same as tens, hundredths being the hundreds. Secondly, children also think that when number zero is added to the right of the decimal point then the value of that number has automatically changed. In this case, it is important to show learners that the zero is used as a place value holder and nothing changes in the number in terms of value. In addition, some learners by just looking at the total number of decimal numbers they would quickly conclude that such number with more decimal number is greater than a number with a few decimal numbers. It is therefore important for teachers to use place value charts and spike abacuses to remove such misconceptions.

## Developing the concept of decimals

Children develop the concept of decimals before they start formal education. In this task you will analyze instructional materials for the teaching and learning of decimals. You will also explore how learners develop the concept of decimals

## Activity 1 Exploring how learners develop the concept of decimals

1 Discuss ways in which learners may establish prior knowledge on decimals.
2 Report your findings to the whole class.

## Activity 2 Analyse primary schools curriculum to determine the expectations on decimals

1 Research on the expectations on decimals in Standards 5 to 8.
2 Discuss the importance of using decimals in such situations.
3 Report to the class on the findings.

## Activity 3 Exploring activities in the teaching of decimals.

1. Research the meaning of decimal.
2. Identify the methodologies and resources that can be used to model decimals.
3. Present the findings to the class.

## Activity 4 Converting decimals into fractions and fractions to decimals

a. Do some research in the library on converting decimals into fractions and vice versa.
b. Use relevant examples when presenting the results to the class.

## Appropriate methodologies in teaching and learning of the concept of decimals

Teaching decimals requires use of appropriate teaching and learning methodologies and resources. Teachers understanding of the instructional materials and skills on decimals will help learners to iron out errors and misconceptions. In this task, you will explore activities and appropriate teaching and learning methodologies for teaching the concept of decimals.

## Activity 1 Analysing learners'

 misconceptions, errors and strategies when solving decimals1 Analyze learners' work on decimals and identify errors and misconceptions made.
2 Suggest possible ways of correcting the errors and the misconceptions made by the learners.
3 Present your work to the class for evaluation.

## TIP

Collect learners' mathematics exercise book (i.e. work book for mathematics) from the demonstration or a nearby school for any class between standards 5 to 8.

## Application of appropriate assessment methodologies in the teaching and learning of decimals

Assessment supplements the teaching and learning process. In this task, you will explore appropriate assessment methodologies in the teaching and learning of the concept of decimals.

## Activity 2 Discussing ways of assessing learners on decimals

a. Suggest ways of assessing the learners on the decimals.
b. Formulate assessment tools which would be used to assess the learners on decimals.
c. Share your ideas to the class.

## Summary

The concept of decimals is well understood when learners have a good understand of fractions. Teachers need to use appropriate methods and resources to fully assist learners develop the concept of decimals and its application in everyday life. Regular assessment of learners on decimals will help you to give appropriate feedback.

## Reflection and assessment

1 What is a recurring decimal?
2 The place value chart below shows a decimal number 325.028 , copy and complete the chart.

| $10^{\mathbf{z}}$ |  |  |  |  |  | $10^{-\boldsymbol{x}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Hundreds | Tens | Ones |  | • tenths |  | thousandths |
|  | T | O |  |  | h |  |
| 3 | 2 | 5 |  | 0 | 2 | 8 |

3 Explain how you can teach learners that $\frac{22}{7}=3.142$

## Glossary

\(\left.$$
\begin{array}{cl}\text { Decimal point } & \begin{array}{l}\text { a point between a } \\
\text { whole number and } \\
\text { a fraction. }\end{array}
$$ <br>
a digit to the right <br>

Decimal place the decimal\end{array}\right\}\)| point. |
| :--- |
| the position of a |
| Decimal value |
| digit that indicates |
| its value. |

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## TOPIC 6 Teaching of approximation and estimation

Time 2 hours

## Introduction

Accuracy is a very important tool in all fields that deal with measurement of quantities. Accuracy by definition refers to how close a measured value is to the actual (true) value. Accuracy in measurement is achieved through estimation and approximation. In our daily life there is no single field where no measurements of quantities are involved. As such estimation and approximation are important mathematical concepts. Estimation and approximation are useful in field like medicine when prescribing medicine, in sports when determining time taken in race and in many other fields.

## Success criteria

By the end of this topic, you must be able to:

- analyze how learners develop the concept of approximation and estimation.
- apply appropriate methodologies in teaching and learning the concept of approximation and estimation.
- use appropriate assessment methodologies in the teaching and learning of approximation and estimation.


## Background information

Approximation and estimation are two mathematical concepts which are often not fully understood by many student

Teachers and learners. The two concepts are confusing to most learners because they are used interchangeably. To most student teachers the two mathematical concepts are the same. They feel that approximation could mean estimation. However, the two mathematical concepts are different but related. Approximation means making measurements to the nearest value depending on level of accuracy of measuring devices. On the other hand estimation is guessing an amount of quantity according to experience that is close enough to the real or right amount. Estimation is a tool for processing approximation. Estimation is not trying to get an exact response to a mathematical question but to get a response which is good enough for the question. Estimation either provide a response which is either wrong or right but close enough to the required response.

Approximation is achieved by rounding numbers to either decimal fractions or significant figures. The skill of approximating lies on the understanding of closeness of two points. In most cases student teachers do not have challenges in dealing with rounding off numbers based on the decimal places. However, challenges do emerge with the significant figures. This is generally because student teachers as well as learners think rules for rounding off decimal fractions automatically apply to significant figures.

## Developing the concepts of approximation and estimation

Understanding and building knowledge on what learners already know about approximation and estimation provides the teacher with facts to use during lesson planning. Mastery of the skill to probe learners on their understanding of concepts, skills, words as well as games that are associated with approximation and estimation is important.

Activity 1 Establishing learners' prior knowledge and skill in approximation and estimation

1 Analyse ways in which learners prior knowledge on approximation and estimation can be elicited during the lesson.
2 Discuss the ways with group members.
3 Present the consolidated findings to the class for evaluation.

Activity 2 Discussing the importance of approximation and estimation
1 Consider the case study below: Mr. Phiri wanted to buy some bags of fertilizers in town. He planned to purchase each bag at K125.00.
When he went to town, each bag of fertilizer was selling at K120.00.
i Did Mr Phiri use the concept of estimation effectively?
ii Was the estimation done by Mr. Phiri a good or bad one?
2 Cite other examples or situations in our daily life in which approximation and estimation are used.
3 Report to the class on the findings.
(an illustration showing a girl pointing at a netball goal post telling her friends that the post is about 4 meters in height)

Activity 3 Determine the primary school curriculum expectations on approximation and estimation in standards 6 to 8

1
xamine the following teaching documents:
i. Teaching Syllabus.
ii. Teachers Guides.
iii.Learners' Book.
stablish the following in the teaching documents that you have examined:
a. The scope and sequence on approximation and estimation.
b. The teaching and learning resources that have been used on approximation and estimation.
c. The complexity of problems on approximation and estimation

3 Report your finding to the whole class for evaluation.

Activity 4 Exploring activities in the teaching of approximation and estimation
1 Research on different strategies and resources you would use when teaching the meanings of approximation and estimation
2 Formulate questions that are relevant in introducing approximation and estimation to learners.
2 Explain how each strategy would be used.
3 Share and critique the planned activities

Activity 5 Exploring concepts and skills in the teaching of approximation and estimation

1 In groups, generate questions on the following concepts:
a. rounding off

E
b. significant figures
c. estimating quantities

2 Create tasks and present to the class how you would teach each question to learners
3 Evaluate the work and refine the tasks

approximation and estimation
Teaching approximation and estimation involves use of suitable teaching and learning methodologies and resources. In this task, you will explore activities and appropriate teaching and learning methodologies for teaching the concept of approximation and estimation.

## Activity 1 Analysing learners'

 misconceptions, errors and strategies on approximation and estimation4 Conduct a research to collect primary school learner's work on either approximation or estimation.
5 Analyse learners' work to identify errors and misconceptions made.
6 Analyse learners work to find out strategies that are used is solving problems on approximation or estimation
7 Suggest possible ways of correcting the errors and the misconceptions made by the learners.

8 Present your work to the class for evaluation.

## Activity 2 Modifying activities on approximation and estimation

1 Plan a lesson on one concept of approximation and estimation.
2 Conduct a micro lesson.
3 Evaluate the lesson and explore possible ways to modify tasks in the lesson to suit inclusiveness and large classes.
4 Present your work to the class for evaluation.

## Application of appropriate assessment methodologies in the teaching and learning of approximation and estimation

Assessment enhance teaching and learning process. Adequate and relevant assessment provides accurate and timely feedback to the teacher on how his or her learners are progressing in the concept. In this task, you will explore appropriate assessment methodologies in the teaching and learning of the concept of decimals.

## Activity 1 Discussing ways of assessing learners on approximation and estimation

1. In groups, develop the following assessment tools for approximation and estimation
a. checklist on estimation
b. oral questions
2. Present the work to the class for analysis
3. Modify the tools to suit learners of different abilities

## Summary

Approximation involves giving a measurement to the nearest value as possible and estimation is guessing a measurement of which it's subjective. The two concepts are used in everyday life situations like estimating age of a person looking at his her physical features. Teaching approximation and estimation requires that the teachers are well conversant with the content and pedagogy of approximation and estimation.

## Reflection and assessment

1 Explain a step by step procedure you would follow when teaching the following to standard 5 to 8 learners

1. round off 23.0826 to 2 decimal places
2. write 76294 to 3 significant figures
2 Which other teaching and learning resource would you in the teaching of rounding off numbers?

## Glossary

Significant figures digits in a number that hold an important place value after rounding off.
Approximation making measurements to the nearest value depending on level of accuracy of measuring devices.
Estimation guessing amount of quantity according to experience

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## TOPIC 7 Teaching rate, ratio and proportion

Time 10 hours

## Introduction

Rate, ratio and proportion is one of the topics in Primary School Mathematics curriculum. Good understanding of this topic will help you in many real life situations, for example in farming when determine how much produce may be realized over a certain period of time, when determining distance to travel over a given time and when sharing quantities. Knowledge of rate, ratio and proportion is also necessary for learners in understanding other mathematical concepts such as fractions, percentage, decimal places, and profit and loss.

In the topic, you will analyse how learners develop the concepts of rate, ratio and proportion and application of appropriate teaching, learning and assessment methodologies in the teaching and learning of the concept of rate, ratio and proportion.

## Success criteria

By the end of this topic you must be able to:
1 analyze how learners develop the concepts of rate, ratio and proportion.
2 apply appropriate methodologies in teaching and learning the concept of rate, ratio and proportion.

3 apply appropriate assessment methodologies in the teaching and learning of rate, ratio and proportion.

## Background information

Rate and ratio are concepts that produce a set of ordered pairs of relationships. They both involve comparing of two quantities. However, the two differ in that rate compares quantities of different units (e.g. 20 eggs per day) while ratio compares two quantities of the same kind or units (e.g. a group of three girls compared to a group of five girls). Proportion on the other hand is the equality of two ratios. In other words, it is a mathematical statement that shows two or more equal ratios. For example $\frac{5}{6}=\frac{10}{12}=\frac{30}{36}$ is a proportion. The numerator for the two ratios have the same units so too the denominators.

The concept of rate, ratio and proportion is well understood by learners when teachers use appropriate teaching, learning and assessment methodologies that involve learners in the lesson.

Most learners in primary school have problems to understand questions on proportion involving three ratios,
especially when one item is compared to two other differently. Teachers' good pedagogical content knowledge on the topic will assist learners minimize their challenges in understanding the topic.

## Developing the concepts of rate, ratio and proportion

Learners have different levels of background knowledge for the topic of rate, ratio and proportion. These concepts are covered in senior classes in the Primary school curriculum in Malawi. Before starting teaching the topic, it is important to know learners' levels of competences so that appropriate teaching, learning and assessment methodologies could be employed.

This task focuses on eliciting learners' prior-knowledge on rate, ratio and proportion as well as discussing importance of rate, ratio and proportion

## Activity 1 Eliciting learners' prior knowledge on rate, ratio and proportion.

1 Explore situations in life from which learners would develop the ideas of rate, ratio and proportion.
2 Explain how each situation is linked to rate, ratio and proportion.

Activity 2 Discussing importance of rate, ratio and proportion in everyday life
1 With reference to activity 1 part a. describe the importance of rate, ratio and proportion in daily life.

## Application of appropriate methodologies in teaching and learning of the concept of fractions.

Teaching rate, ratio and proportion to different classes and levels of learners requires use of appropriate teaching and learning methodologies. Many teachers approach these concepts differently depending on their pedagogical content knowledge. Teachers need to understand the instructional materials and skills expected at each level in the primary school curriculum. In this task, you will be introduced to rate, ratio and proportion. You will also explore activities and appropriate teaching and learning methodologies for teaching rate, ratio and proportion.

## Activity 1 Examining the primary school curriculum on rate, ratio and proportion

1 Examine the teaching syllabus, teachers' guides and learners' books for standard 6 to 8 n ratio, rate and proportion.
2 Determine expectations on rate, ratio and proportion from standards 6 to 8 .
3 Report your work to the class.

## Activity 2 Exploring activities in the

 teaching of rate, ratio and proportion1 Explore activities you would use to introduce the concept of rate, ratio and proportion.
2 Share with the class the meaning of rate, ratio and proportion based on the activities.
3 Explain the relationship between rate, ratio and proportion.

## Activity 3 Exploring types of proportions

1 Analyse several questions on proportion and classify them.
2 Identify the types of proportion.
3 Describe the difference between the types.

Activity 4 Applying problem solving skills in teaching learners rate, ratio and proportion
1 Consider the following questions:
i A motorist drives 100 km in 1hour. Express the speed in meters per second
ii At a Teacher Training College, there is enough food to feed 240 student teachers for 15 days. After 3 days, 20 students withdrew from the college. How many days would it take for the remaining food to last?
iii Share 810 exercise books among three learners, Tawina, Kondwani and Mphatso so that Tawina gets half as much as Kondwani and Mphatso gets half as much again as Tawina.How much does each one get?
2 Using different strategies, show how you can solve the above questions.

3 Generate several questions on rate, ratio and proportion that require application of problem solving skills.


## Application of appropriate assessment methodologies in the teaching and learning of rate, ratio and proportion

Assessment complements the teaching and learning process. In this task, you will explore appropriate assessment methodologies in the teaching and learning of the concept of rate, ratio and proportion. This will help you to get necessary feedback from your learners and eventually use appropriate methodologies to support them.

$$
\begin{array}{ll}
\text { Activity } 1 & \text { Analysing learners' } \\
& \text { misconceptions, errors } \\
& \text { and strategies when } \\
& \text { solving rate, ratio and } \\
& \text { proportion }
\end{array}
$$

1 Research on misconceptions and errors learners have on rate, ratio and proportion
2 Discuss the sources and causes the misconceptions and errors.
3 Suggest strategies that you would use to correct learners misconceptions and errors.
4 Share with the class.

## Activity 2 Discussing ways of assessing learners on fractions

1 Prepare a lesson plan on rate, ratio and proportion and micro-teach it.
2 Suggest assessment tools that you would use to assess learners' competency.
3 Explain how each tool would be used.

## Tip

You may collect learners' written work from the school.

## Summary

The topic has presented work on how student teachers would develop the concepts of rate, ratio and proportion with their learners. It has also covered work on appropriate methodologies which student teachers may use during the teaching of rate, ratio and proportion. It further contains work on the approaches which student teachers would use in assessing the learners in the primary school on rate, ratio and proportion.

## Reflection and Assessment

1 What challenges may be encountered when teaching learners on rate, ratio and proportion?
2 Using an example, explain the difference between unitary and ratio factor method
3 Formulate a check list with three items which can be used to assess the learners after teaching proportion.
4 If the ratio of teacher to learner in a school is 1 : 28 and there are 560 learners in the school, how many teachers are there in the school?

5 The distance between two towns is 330 km . If a train takes 5 hours to cover the distance, what is the average speed of the train?
6 How would you use 5 objects to introduce ratio to learners.
7 A girl's age is 12 years, her father's age is 48 years. What will be the ratio of their age in 4 years?
8 Simplify the ratio $2 \frac{1}{2}: 1 \frac{1}{4}$.
9 Share 12 mangoes to John and Talandira in the ratio of $1: 3$.
10 If 9 men can paint a building in 21 days, how long would 7 men take to paint the same building?
11 If 3 men can do a job in 4 days, working 10 hours a day. How many days will it take 8 men working for 6 hours a day?

## Glossary

Rate comparison of quantities of different types or a
correspondence between two different measures.
Ratio comparison of two quantities of the same kind.
Proportion equality of two ratios.

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## TOPIC 8 Teaching of capacity and volume

volume is the total space occupied by a substance. It is measured in cubic centimeter or meter.

Most learners in primary school have challenges in differentiating the two concepts. Involving learners in measuring capacity and volume helps them in establishing a strong relationship between the two concepts and iron out the misconceptions. The concept of capacity and volume is also well understood when teachers apply appropriate teaching, learning and assessment methodologies.

## Developing the concept of capacity and volume

It is essential to establish learners' priorknowledge on capacity and volume. This may assist teachers in terms of planning for their lessons.

This task focuses on eliciting learners' prior-knowledge on capacity and volume as well as discussing importance of capacity and volume

In this task, you will explore learners' prior knowledge on capacity and volume as well as discussing importance of capacity and volume in real life.

Activity 1 Establishing prior knowledge and discussing importance of capacity and volume
1 Think of situations in life from which learners would develop the idea of capacity and volume.
2 Explain how each situation is linked to capacity and volume.
3 Discuss the importance of capacity and volume in life.
4 Analyse primary school curriculum materials to determine the expected level of learners' competencies.

## Application of appropriate methodologies in teaching and learning of the concept of capacity and volume

Teaching capacity and volume requires use of appropriate teaching and learning methodologies. Capacity and volume is first introduced using non standard units.

In this task, you will also explore activities and appropriate teaching and learning methodologies for teaching capacity and volume.

## Activity 1 Exploring activities for introducing capacity and volume

1 Explore strategies that you would use to introduce the concept of capacity and volume.

2 Explain how each strategy would be used.

Activity 2 Discussing activities for measuring capacity and
volume using non standard units

1 Explore the meaning of non standard units.

2 Identify resources that would be used as non standard units for measuring capacity and volume
3 Explain how each resource is used as non standard unit for measuring capacity and volume.
4 Describe the advantages and disadvantages of using non-standard units for measuring capacity.

Activity 3 Exploring activities for introducing capacity and volume using standard units
1 Think of strategies that would be used when introducing standard units for capacity and volume.
2 Explain how each strategy would be used.

3 Share with the class.

## Activity 4 Establishing the

 relationship between capacity and volume1 Develop activities to show that 1 litre is equal to $1000 \mathrm{~cm}^{3}$
2 describe other relationships between capacity and volume
3 Share with the class.

$$
\begin{aligned}
& \text { Tips } \\
& \text { - The concepts should be taught } \\
& \text { separately } \\
& \text { - Research more the theory of } \\
& \text { displacement } \\
& \text { - Note: } 1 \mathrm{~cm}^{3}=1 \mathrm{ml}, 1000 \mathrm{~cm} 3=1 \\
& \text { litre, } 1 \text { litre }=1000 \text { millilitres }
\end{aligned}
$$

## Application of appropriate teaching, learning and assessment methodologies

## Activity 1 Discussing ways of assessing learners on capacity and volume

1 Suggest assessment tools that you would use to assess learners' competency when teaching volume of irregular objects.

2 Explain how each tool would be used.

## Summary

Capacity is the ability of a container to hold something. Volume is a measure of space occupied by an object. Teaching capacity and volume starts with nonstandard units. Standard unit of volume is $\mathrm{m}^{3}$. Cubic centimetre $\left(\mathrm{cm}^{3}\right)$ is commonly used when measuring volume. Learners should learn capacity and volume because it helps them to be competent in life. These concepts should be taught using appropriate teaching, learning and assessment methodologies. Relationship between capacity and volume is that 1 litre $=1000 \mathrm{~cm}^{3}$ and 1 ml $=1 \mathrm{~cm}^{3}$.

## Reflection and assessment

1 Find the volume of a rectangular room with length 8 m , width 3 m and height 4 m .
2 A pail is 20000 ml , how many pails can be filled from a tank containing 150 litres of water?

3 A rectangular tank is 5 m long, 3 m wide and 2 m deep. How many litres
of water does the tank hold when it is one quarter full?
4 Write down a step by step procedure to show learners that volume of a cuboid is length $x$ width $x$ height
5 Explain any two advantages of standard unit over non standard units when finding capacity.
6 2. Show how you can help learners to convert 2.56 litres to ml .

7 What is the difference between 10 cm cube and 10 cubic cm ?

8 Calculate the volume of a cylindrical tin whose diameter is 5.6 cm and height is 8 cm .

9 A circular metal block 60 cm high and radius 14 cm has a cylindrical hole at the center of radius 7 cm parallel to the longest edge. Calculate the volume of the metal remaining after the hole has been cut.

## Glossary

Capacity ability of a container to hold or keep something
Volume a measure of space occupied by an object.
Cube a block with six square faces.
Cuboid a block with six rectangular faces.
Non- standard unit these are traditional way of measuring substances or something.
Standard unit modern units of measurement (international/ conventional way of measuring.

Volume the space occupied by an object.

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