Initial Primary Teacher Education

Mathematics

Module 4

Malawi Institute of Education

Initial Primary Teacher Education

Mathematics

Module 4

Malawi Institute of Education

First edition 2018

Foreword

Education is the lifeblood of the nation. It is a prerequisite for individual, community and national development. Education prepares learners to play their roles effectively to promote and sustain a country's socio-economic development. Parents or guardians desire that their children develop into adults with sound minds and healthy bodies through the acquisition of appropriate knowledge, skills and desirable attitudes to enable them to live productive and happy lives.

Education should, therefore, help learners to develop high standards of conduct, attitudes, judgment and a sense of responsibility. Student teachers have to be well prepared in order to be able to take this responsibility of teaching children effectively.

The provision of quality education is based on many factors and a good quality of teachers is one of them. Teachers play a central role because they are the key source of knowledge, responsible for facilitating the learning process and act as role models for the learners.

The function of initial teacher education in Malawi is to prepare student teachers in their aspiration of becoming teachers of high quality. This is achieved by helping the student teachers to acquire the right knowledge, skills and competences to enable them to effectively teach children. In view of this, the Initial Primary Teacher Education curriculum has been reviewed to ensure that student teachers who graduate from this programme are well trained and prepared for their profession.

The process and implementation of this review has been guided by the Teacher Education Philosophy which states as follows:

'To produce a reflective, autonomous, lifelong learning teacher, able to display moral values and embrace learners' diversity.'

It is therefore hoped that Teacher Training Colleges will find this curriculum effective in helping the student teachers to build a solid foundation in their teaching profession.

Executive Director
Malawi Institute of Education

Acknowledgements

Writers

Getrude Jumbe	-	Blantyre Teachers' College
Eneya Phiri	-	Blantyre Teachers' College
Paschal Kayange	-	Karonga Teachers' College
Gabriel Chamdimba	-	Machinga Teachers' College
Bruno Chikopa	-	Machinga Teachers' College
Adhija Nangoma	-	Maryam Girls Teachers' College

Contents

Introduction

The purpose of primary teacher education is to produce and continually develop competent and responsive teachers who effectively deliver quality education to all learners under prevailing conditions and demands in primary schools and promote their desire for life-long learning. IPTE endeavors to educate teachers in sufficient numbers, continually develop their professionalism so that they are able to effectively and efficiently deliver quality and relevant education to primary school learners.

National goals for primary teacher education

The national goals of primary teacher education in Malawi are to produce teachers who are:
- academically well-grounded and professionally competent
- flexible and capable of adapting to the changing needs and environment of the Malawian society
- capable of adhering to and maintaining the ethics of the teaching profession imaginative in adapting, creating and utilising locally available resources suitable for the needs of their learners.

Rationale

Mathematics education aims at developing student's critical awareness of mathematical concepts and their relationships and how these are used for solving practical problems in a social, environmental, cultural and economic context.

At an early stage, the learners will be able to count and carry out basic mathematical operations. At a later stage, the learners will be able to make inferences using manipulated data and to apply mathematics for solving practical problems in their daily life.

Teacher education philosophy

The following has been the guiding principle during the design, development and implementation of the IPTE curriculum.

To produce a reflective, autonomous, lifelong learning teacher, able to display moral values and embrace learners' diversity.

IPTE programme structure

The duration of the teacher education programme is two years. The general outlook of the two years is as follows:

Year 1			Year 2		
Term 1	**Term 2**	**Term 3**	**Term 1**	**Term 2**	**Term 3**
In college, learning subject content with a special focus on methods for lower classes	In college, learning subject content with special focus on methods for upper classes	Out in teaching practice schools, practising teaching mainly in the lower classes	Out in teaching practise schools, practising teaching mainly in the upper classes	In college, with special emphasis on reflection, inclusion and further practice on teaching methods	In college, with special emphasis on subject content, policies and frameworks

Unique features

The features of the reviewed curriculum are as follows:
- The curriculum design is based on reflective and practice principles.
- Early grade teaching methodologies are distinct.
- The delivery of the subject content follows the modular approach.
- Student teachers will be allowed to practise teaching both in the lower classes (Standards 1 to 4) as well as in upper classes (Standards 5 to 8).
- Cross cutting issues, such as Assessment for Learning, Information Communication Technology, Inclusive Education and Critical Thinking, are integrated.

IPTE subject matrix

The new curriculum has adopted the reflective practitioner model of teacher education which connects reality and theory and integrates content and pedagogy in teaching and learning. In this structure, student-teachers will be in college for terms 1 and 2 of year 1 and be in primary schools for teaching practice in the third term of year 1 and first term of year 2. Students will be back to college in terms 2 and 3 of year 2 to continue learning subject content, reflecting on their experiences during teaching practice and then wind up their studies.

This curriculum has adopted a modular design and contains eleven subjects. These are Chichewa, English, mathematics, education foundation studies, agriculture, social studies, life skills, science and technology, expressive arts, religious studies and human ecology. In this modular design, a set of units with related content forms a module in a subject. A module consists of 40 contact hours.

Although the programme is modular, four subjects: Chichewa, English, mathematics and education foundation studies will be offered throughout the two years while the remaining subjects will be spread across the two years.

IPTE outcomes based curriculum

An outcomes-based curriculum is focused on students' achievement. To achieve the outcomes, the students are introduced to new knowledge in the context of their existing knowledge so that they can develop new understandings. Therefore, the process of learning is integral to the final product. These products are the outcomes, which student teachers achieve in terms of teaching competencies and must be clearly stated before they begin teaching. The achievements made at college, however, will only be seen to be truly beneficial when student teachers transfer the knowledge, skills and desirable attitude beyond college and view learning as a life-long process. This is considered essential to keep pace with the transition from college to practical classroom experiences.

There is need for student teachers to acquire knowledge, skills and desirable values and attitudes to enable them to implement the national primary curriculum. To that end, student teachers should be fully conversant with foundation studies and other subjects taught in schools.

Learning areas and core elements

A learning area is an organised body of the required knowledge, skills, values and desirable attitudes that serve as a foundation for future learning. Each learning area has a rationale from which core elements are derived. The IPTE curriculum comprises eleven learning areas namely agriculture, science and technology, mathematics, expressive arts, Chichewa, English, foundation studies, social studies, life skills, religious studies and human ecology. Each learning area has four or more core elements.

Teacher education core element outcomes

Teacher education core element outcomes are descriptions of the competencies to be acquired by the student teacher for successful teaching.

Progression of learning areas into subjects in the primary senior phase

Student teachers should know that during infant and junior phases of primary school, teaching and learning will centre on learning areas instead of isolated subjects. However, the curriculum will revert to subjects in the senior phase in line with the secondary school curriculum. For this reason, student teachers will study both learning areas and subjects.

IPTE assessment procedures

In Outcomes-Based Education (OBE), assessment is a significant part of the teaching and learning process. The main purpose of assessment is to facilitate learning by constant monitoring of the progress of individual learners. The process is on-going and it uses clearly defined criteria with a variety of tools, methods and techniques in different situations and contexts. This helps to gather valid and reliable information on the learners' achievement of outcomes.

Assessment in primary teacher education in Malawi comprises two major components: continuous and summative assessment. Both modes involve assessment tasks that measure the student teachers' achievement of knowledge, skills, values and attitudes. These tasks include oral presentations, practical tasks, reports, research, tests and examinations.

In the reviewed curriculum, the weighting of continuous assessment in the final grade will be *60% continuous assessment* and *40% summative assessment.*

The continuous assessment will comprise:
- two assignments based on each module
- end of module examinations excluding terms 2 and 3 of year 2
- teaching practice grades
- school experience journal grade

While the summative assessment will comprise:
- moderated grade from teaching practice in term 3 of year 1 and term 1 of year 2
- national examinations to be administered in term 3 of year 2 based on the modules of terms 2 and 3 of year 2

Core elements and their outcomes

Theories, concepts and issues in the teaching and learning of mathematics

The student teachers will be able to demonstrate an understanding of theories, concepts and issues in the teaching and learning of mathematics and how they will apply these to their teaching of mathematics in primary school.

Number concepts and operations

The student teachers will be able to demonstrate appropriate pedagogical knowledge in the teaching and learning of number concepts and operations to enable primary school learners use numbers and their relationships in everyday life

Measurement

The student teachers will be able to demonstrate appropriate pedagogical knowledge in the teaching and learning of measurement to enable primary school learners apply appropriate measurement skills in everyday life.

Data handling

The student teachers will be able to demonstrate appropriate pedagogical knowledge in the teaching and learning of data handling to enable the primary school learner analyse and interpret data for decision making by using graphs and tables in relation to everyday life.

Space and shape

The student teachers will be able to demonstrate appropriate pedagogical knowledge in the teaching and learning of space and shape to enable primary school learners use skills of space and shape in everyday life.

Accounting and business studies

The student teachers will be able to demonstrate appropriate pedagogical knowledge in the teaching and learning of accounting and business studies to enable primary school learner acquire basic knowledge and skills on financial management.

Patterns, functions and algebra

The student teachers will be able to demonstrate appropriate pedagogical knowledge in the teaching and learning of patterns, functions and algebra to enable the primary school learner use algebraic language and develop skills to solve textual problems.

Summary of topics for the term and time allocation

Term 3		
Topic	**Allocated time in hours**	**Core element**
Teaching of money	6	Accounting and business studies
Teaching of postal and bank services	2	
Teaching of commission and discount	4	
Teaching taxes and premiums	3	
Teaching of simple and compound interest	4	
Teaching of simple accounts	1	
Teaching of patterns	6	Patterns, functions and algebra
Teaching of algebraic expressions	4	
Teaching of equations	6	
Teaching of inequalities	4	

Teaching of money in Standards 1 to 4

Time 6 hours

Introduction

Money is very important because it provides a means of expressing the value of an object. Exchanging of goods and services is usually based on the value expressed in terms of money. Learners begin school with experiences of money. They have used coins and bank notes to buy things such as sweets, sugar and salt from shops and markets. Knowledge and skills about money will help learners make transactions involving money properly in everyday life.

In this unit, you will analyse how learners develop the concept of money and discuss how to teach and assess learners on the topic effectively.

Success criteria

By the end of this topic, you must be able to:

- analyse how learners develop the concept of money
- apply appropriate methodologies when teaching and learning money
- use appropriate assessment methodologies in the teaching and learning of money

Background information

Money plays an important role in everyday life. It is a medium of exchange. Learners develop the concept of money in their home where they use coins and bank notes. Teaching of money involves helping learners in recognising, naming, writing and finding the value of money in terms of other denominations. Figure 1.1 shows coins and bank notes.

Fig 1.1: Coins and bank notes (1t, 2t, 5t, 10t, 20t, 50t, K1, K5, K10, K20, K50, K100, K200, K500, K1000 and K2000)

Therefore, student teachers should acquire suitable knowledge and skills for teaching and learning of money, that is, give examples using real life situations. The student teachers should also use a variety of activities with a shopping scene during basic operations such as addition, subtraction, multiplication and division. This will promote learners' understanding of money in real life contexts.

Task 1   Developing the concept of money

Learners come to school with lot of experiences of money. Therefore, teaching money should take into account what learners already know. In this task, you will discuss learners' prior knowledge of money.

Activity 1   Discussing learners' prior knowledge and importance of money

1   Discuss learners' prior knowledge on money.
2   Explore situations in everyday life that help children learn about money.
3   Discuss the importance of learning money to primary school learners.
4   Explain how money is applied in everyday life.
5   Share your ideas with the class.

Activity 2   Discussing primary school curriculum expectations for teaching money

Analyse primary school instructional materials such as syllabuses, teacher's guides, learners' books and other relevant sources and carry out the following:
1   identify the concepts and skills of money that are taught in primary school. Present your work in a table

Class	Concepts

2   determine the expected knowledge and skills that learners would acquire after learning money

3 report your findings to the class for discussion

Task 2 Appropriate methodologies for teaching and learning of money

Primary school learners must be engaged in a number of activities using relevant resources for them to develop meaningful knowledge and understanding of money. In this task, you will explore appropriate methodologies in the teaching and learning of money.

Activity 1 Teaching how to describe features on coins and bank notes

1 Identify teaching and learning resources you would use in the teaching and learning of money.
2 Discuss how you would teach the following using the identified resources:
   - describing features on coins and banknotes
   - writing of money
   - expressing values of money
   - performing basic operations on money
3 Share your ideas with the class for discussion.

Tips
- You could use resources such as real money and dummies.
- Modify the activities to meet learners with diverse education needs

Task 3 Appropriate assessment methodologies in the teaching and learning of money

Assessment provides feedback to both the teacher and learners about how well teaching and learning progress. In this task, you will explore appropriate assessment methodologies in the teaching and learning of money.

Activity 1 Analysing learners' strategies, misconceptions and errors on money

1 Analyse strategies that learners use when carrying out various activities on money such as:
   - expressing value of money
   - performing basic operations on money
2 Identify learners' misconceptions and errors on money.
3 Discuss causes of the misconceptions and errors.
4 Suggest effective instructional approaches that you would use to address the misconceptions and errors
5 Report your findings to the class for discussions.

Activity 2 Discussing appropriate ways of assessing learners on money

1 Discuss appropriate teaching, learning and assessment methods on money.
2 Prepare a lesson plan on any concept on money.
3 Peer teach the lesson

4   Evaluate the lesson by focusing on assessment methods used, such as oral questions, observation and written work.

Reflection and assessment

1   Describe learners' misconceptions on money.
2   Develop appropriate assessment tools that you can use in the teaching and learning of money.
3   Describe how you would use a shopping scene to introduce basic operations on money

Summary

In this topic, you have learnt about the teaching of money. Concepts and skills when teaching money include identifying features on coins and bank notes, expressing values of money in terms of other denominations and performing basic operations on problems involving money. When teaching concepts on money, you need to use various teaching, learning, assessment and resources as well as varying teaching, learning and assessment methods. This helps learners understand the concepts clearly.

Glossary

Price tag	:	label indicating price of an item
Shopping scene	:	a role play where by learners practise buying and selling
Price list	:	outline of prices of items

References

Ministry of Education Science and Technology (2008). *IPTE numeracy and mathematics, students' book.* Domasi: MIE.

Ministry of Education Science and Technology (2008). *Teachers' guides for Standards 1 to 4.* Domasi: MIE.

Further reading

"Strategies to reduce errors"
https://childcareta.afc.hhs.gov/sites/defualt/files/public

10 Common misconceptions about money
*https://www.yahoo.com/news/10-common-misconception-money*

TOPIC 2 Teaching of postal and bank services

Time 2 hours

Introduction
How did you get information about your selection to pursue studies at teacher training college? May be some of you got it through phone calls, letters, e-mails or fax. The act of offering services of sending and receiving information, parcels and money is termed as postal services. There are different institutions which provide postal services in Malawi. For example, Post Office, FEDEX, DHL ,Pony and G4S. In addition, some institutions offer bank services such as keeping people's money, lending money to individuals and organisations and sending and receiving money. Some of these institutions include National Bank of Malawi, Standard Bank of Malawi and currently, Airtel Money, Zoona, Telecom Networks Malawi (TNM) Mpamba and Village Banks. Figure 2.1 shows a bank view and postal office face.
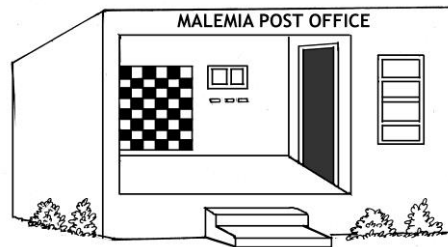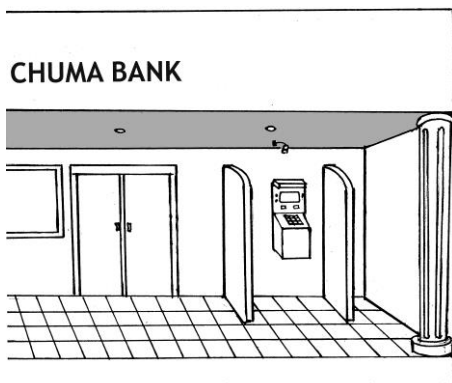
Fig 2.1: Bank view and postal office face

In this topic, you will learn about learners' understanding of postal and bank services and discuss appropriate teaching, learning and assessment methodologies of postal and bank services. Knowledge and skills about these services will help you teach the topic competently to primary school learners so that they use the services appropriately in their everyday life.

Success criteria
By the end of this topic, you must be able to:
- analyse learner's understanding of postal and bank services
- apply appropriate methodologies when teaching postal and bank services

5

- apply appropriate assessment methodologies in the teaching and learning of postal and bank services

Background information

People use postal and bank services in their everyday life. Postal services include delivery of items such as letters and parcels. They also include sending and receiving money. People pay for these postal services. The cost depends on the mass of an item or the amount of money sent. This cost is termed as commission. Commission is a charge levied against a service rendered which is calculated as a percentage of the value of the money sent or for sending items.

Banks are essential because they offer services such as keeping people's money safe, lending money to individuals and organisations. Banks also provide services like paying bills for utilities and transferring money from one person's account to another. Figure 2.2 shows bank slips.

Fig 2.2: Bank slips

6

When teaching postal and bank services, learners should practise filling in postal forms, withdraw forms, cash and cheque deposit slips. They should also calculate charges of sending items and parcels.

Task 1 Learners' understanding of postal and bank services

Learners are involved in sending and receiving items and parcels through post office and banks in their communities. Through that, learners acquire some knowledge of charges that both post office and banks charge for offering such services. Therefore, it is important to elicit learners' prior knowledge of postal and bank services for effective teaching and learning of the topic.

Activity 1 Discussing learners' prior knowledge and its application in everyday life

1 Discuss learners' prior knowledge of postal and bank services.
2 Identify postal and bank services within your community.
3 Explore situations in everyday life that help children learn about postal and bank services.
4 Discuss the importance of learning postal and bank services to primary school learners.
5 Explain how postal and bank services are used in everyday life.
6 Present your work to the class

Activity 2 Discussing primary school curriculum expectations for postal and bank services

Analyse primary school instructional materials such as syllabuses, teacher's guide, learners' books and other relevant sources and carry out the following:

1 identify the concepts and skills of postal and bank services that are taught in primary school. Present your work in a table

Class	Concepts

2 determine the expected knowledge and skills that learners would acquire after learning postal and bank services
3 report your findings to the class for discussion

Task 2 Appropriate methodologies for teaching and learning of postal and bank services

Teaching of postal and bank services requires use of appropriate teaching, learning and assessment methodologies. Teachers need to understand the instructional materials and skills expected in the national primary curriculum in order to teach learners effectively. In this task, you will explore appropriate teaching and learning methodologies for teaching and learning of postal and bank services.

Activity 1 Exploring activities in teaching and learning of postal services

1 Identify resources that you would use in the teaching and learning of postal services.
2 Discuss how you would use the identified resources to teach each of the following:
   - postal and cash transfer
   - calculating charges on postal services
3 Present your work to the class for discussion.

Activity 2 Exploring activities in the teaching and learning of bank services

1 Identify resources that you would use in the teaching and learning of postal services.
2 Discuss how you would use the identified resources to teach each of the following:
   - identifying bank services
   - types of bank accounts
   - advantages and disadvantages of different bank accounts
   - calculating charges on bank services
   - solving practical problems on bank services
3 Present your work in plenary

Activity 3 Exploring learners' strategies, misconceptions and errors on postal and bank services

1 Anlyse learners' strategies on postal and bank services.
2 Identify learners' misconceptions and errors.
3 Discuss sources of the misconceptions and errors.
4 Discuss effective instructional approaches that you could use to address the misconceptions and errors.
5 Share your work with the class.

Tip
You may use learners' work, video lesson clips and reflect on your experiences from teaching practice on postal and bank services.

Task 3 Appropriate assessment methodologies in the teaching and learning of postal and bank services

Assessment is an integral part of the teaching and learning process. In this task, you will explore appropriate assessment methodologies for the teaching and learning of postal and bank services.

Activity 1   Discussing how to assess learners on postal and bank services

1   Discuss ways of assessing learners on postal and bank services.
2   Develop assessment questions for assessing learners on postal and bank services.
3   Report your work to the class for discussion.

Activity 2   Developing assessment tools for assessing learners on bank and postal services

1   Discuss different tools you could use to assess learners on postal and bank services.
2   Develop a checklist you could use to assess learners on postal and bank services.
3   Report your ideas to the class.

Reflection and assessment

1   With relevant examples, explain the difference between postal services and bank services.
2   Why is it important for primary school children to learn postal and bank services?
3   Discuss how you would teach learners to fill a deposit slip for *Yankho Chitsulo* using the following information: *Yankho Chitsulo* of Box 132, Balaka, wants to deposit ten K1000 notes, nine K500 notes, two K200 notes, one K100 note, two K50 notes, twenty K20 notes, fifty K10 coins and six K5 coins into her savings account number 0134001546.
4   Explain how use of critical thinking methods in the teaching and learning of postal and bank services would help clear learners' misconceptions and errors.

5   The table that follows shows the postage rates for sending different articles.

Item	Mass of item	Postal charges
Letters and small packets	Up to 20g	K 520.00
	Above 20g up to 100g	K 820.00
	Above 100g up to 250g	K1180.00
	Above 250g up to 500g	K1560.00
	Above 500g to 1000g	K3380.00
	Above 1000g up to 2000g	K4680.00
	Per additional step of 1000g	K 520.00
Newspaper	Up to 20g	K 360.00
	Above 20g up to 100g	K 820.00
	Above 100g up to 250g	K 1180.00
	Above 250g up to 500g	K 1560.00
	Above 500g up to 1000g	K 2600.00
	Above 1000g up to 2000g	K 3840.00
	Per additional step of 1000g	K 360.00
Literature for the blind	Up to 7kg	Free
	Additional 1000g	K 820.00

Item	Mass of item	Postal charges
Parcels	Up to 1kg	K 1225.00
	Above 1kg up to 3kg	K 1576.00
	Above 3kg up to 5kg	K 2101.00
	Above 5kg up to 10kg	K 2978.00
	Above 10kg up to 15kg	K 3855.00
	Above 15kg up to 20kg	K 4605.00
	Per additional step of 1000g	K 520.00

a) Using the table, calculate the total cost of sending the following items within Malawi:
   i. a letter weighing 100g
   ii. a newspaper weighing 110g
   iii. literature for the blind weighing 5kg.
b) Explain errors that learners may make in solving the problem.

Glossary
Automatic teller machine (ATM) card   a card used in banks to withdraw from and deposit money to one's account

References

Ministry of Education Science and Technology (2008). *Mathematics learners' book for Standard 7 and 8.* Domasi: MIE.

Ministry of Education Science and Technology (2008). *Mathematics teachers' guide for Standard 7 and 8*. Domasi: MIE.

Ministry of Education Science and Technology (2013). *Malawi primary, secondary and teacher education critical thinking sourcebook for Malawi*. Domasi: MIE.

Hau, S et al. (1995). *Malawi primary teacher education-mathematics*, Zomba. Domasi: MIE.

Malewezi, F., & Mwanjabe, B. (1989). *Secondary mathematics book* 1. Blantyre: Dzuka.

Teaching of commission and discount

Time 4 hours

Introduction

Commission and discount are commonly used in business circles. Sales persons or agents may demand commission if they sell property on behalf of somebody. Buyers may request for discount in markets when they are buying different commodities. Commission and discount might be offered anywhere in the distribution channel.

Similarly, learners might have been involved in buying and selling of items where a commission or discount has been offered. They might have been offered a commission on items they sold or might have been given a discount on items they bought. Learners should be taught commission and discount so that they can apply the knowledge and skills in real life situation.

In this topic, you will analyse how learners develop the concepts of commission and discount and learn how to teach and assess learners on the topic effectively.

Success criteria

By the end of the topic, you must be able to:
- analyse how learners develop the concept of commission and discount
- apply appropriate methodologies in teaching commission and discount

- use appropriate assessment methodologies in teaching commission and discount

Background information

Commission is the money paid for services rendered, or for every item sold. For example, Newspaper sellers are paid money for every Newspaper they sell as commission. Commission is given as an incentive to the seller for selling an item. The more items one sells, the higher the commission they get.

Discount is a deduction from the marked price of goods or services. It is offered for several reasons which include seasonal, promotional and clearing of stock. Prices are reduced with a certain amount to attract customers or to clear stock which has overstayed. For instance, if the price of an item is K5000 and is sold at K4500, the discount is K500. Customers may buy large quantities just because the price has been reduced by K500.

It is, therefore, important that learners should be taught commission and discount so that they meaningfully use the knowledge in everyday life. The teaching and learning of commission and discount should engage learners actively. Hence, teachers should apply appropriate teaching techniques to ensure that all learners take an active role in all lessons. The use of continuous assessment helps teachers adjust the teaching process in order to improve learners' achievement. Modifying the lesson activities ensures

that learners with special educational needs participate actively in the lesson activities.

Task 1   Learners' prior knowledge of commission and discount

Learners are involved in buying and selling of items in their communities where they might experience commission or discount. Therefore, learners come to school with such experiences from where you can build on your teaching.

Activity 1   Discussing learners' prior knowledge of commission and discount and their application in everyday life

1   Explore situations in everyday life that may help children learn about commission and discount.
2   Explain how commission and discount are applied in everyday life
3   Report your findings to class for discussion.

Activity 2   Analysing primary school curriculum expectations for teaching commission and discount

Analyse primary school instructional materials such as syllabuses, teacher's guide, learners' books and other relevant sources and do the following:
1   identify the concepts and skills of commission and discount that are taught in primary school. Present your work in a table

Class	Concepts

2   determine expected knowledge and skills that learners would acquire after learning commission and discount
3   report your findings to the class for discussion

Task 2   Appropriate teaching and learning methodologies for commission and discount

Teaching commission and discount to primary school learners requires use of appropriate teaching and learning methodologies. Teachers need to understand the instructional materials and skills expected in the primary school curriculum in order to teach learners effectively. In this task, you will explore appropriate teaching and learning methodologies for teaching of commission and discount.

Activity 1   Discussing the teaching and learning of commission and discount

1   Discuss how you would introduce commission and discount to learners using the local environment.
2   Discuss how you would teach learners the following:
   -   difference between commission and discount
   -   calculating commission and discount

3 Discuss how you can modify the activities to accommodate learners with diverse needs.
4 Share your work with the class.

Activity 2 Conducting microteaching on commission and discount

1 Prepare a detailed lesson plan on commission or discount.
2 Conduct a micro-lesson.
3 Evaluate your lesson focusing on critical thinking methods used.

Task 3 Using appropriate assessment methodologies in the teaching and learning of commission and discount

Assessment informs teaching and learning. It helps the teacher check learners' achievement levels as well as errors and misconceptions. Therefore, it is important to use appropriate techniques when assessing learners on commission and discount.

Activity 1 Exploring learners' strategies, misconceptions and errors on commission and discount

1 Analyse learners' strategies on commission and discount.
2 Identify misconceptions and errors on commission and discount.
3 Explore possible sources and solutions to the misconceptions and errors.
4 Present your work to the whole class for discussion.

Activity 2 Exploring appropriate assessment methodologies in the teaching and learning of commission and discount

1 Describe suitable ways of assessing learners on commission and discount.
2 Formulate word problems on commission and discount.
3 Prepare a marking key for each question.
4 Present your work to the class for discussion.

Tips

- You may need to take photographs of learners' work or collect their exercises on commission and discount.
- You may also need to observe or watch a video lesson on commission and discount.
- Research in other sources such as online articles and library books to understand learners' misconceptions and errors on commission and discount.
- You may also reflect on your teaching practice experiences on commission and discount.

Summary

Commission is the amount of money paid or received for the services rendered. It is expressed as a percentage or amount of money per number of items sold or certain amount of tambala in every Kwacha.

Discount is the reduction in the marked price of an item. It is found by subtracting selling price from the marked price of an item. Marked price is equivalent to 100 percent while selling price is equivalent to marked price minus discount.

Reflection and assessment

1 Define the following terms:
   a) Agent
   b) Commission
   c) Discount
   d) Marked price
2 State the prerequisite knowledge for teaching commission and discount to primary school learners.
3 a) Alinafe works in a shop. She receives a salary of K25, 500 per month. In addition, she receives a commission of 2.5 percent on sales per month. If her sales for a certain month amounted to K265, 600, how much did she earn altogether in that month?
   1 Discuss how you would teach learners to find a solution to the problem in 3a.

4 Prepare a marking key for each of the following questions:
   a. A sales lady gets a commission of K370.00 on every K4000.00 she makes. If she sold goods for K60, 000.00, find her commission.
   b. 5 baskets of tomatoes are marked K2, 350.00 each. If each basket of tomatoes is sold at K2250.00, find the discount.
   c. Mrs. Gwaza gets a commission of K500.00 for every bicycle sold. If she sold 24 bicycles at K35, 000.00 each, calculate the commission percent.
   d. The selling price of a radio was reduced from K35900 to K32 669. What was the discount percent?
5 Develop a rubric for assessing learners on commission and discount.

Glossary

Discount amount of money deducted from the marked price of an item.

Marked price original price of an item.

References

Ministry of Education, Science and Technology (2009). *Mathematics teachers' guides for Standard 7 and 8*. Domasi: MIE

Ministry of Education, Science and Technology *IPTE (2008). Numeracy and mathematics students' book*. Domasi: MIE

Ministry of Education, Science and Technology (2008). *Initial primary teacher's education teaching syllabus for numeracy and mathematics*. Domasi: MIE

Ministry of Education, Science and Technology *(2013). Malawi primary, secondary and teachers education critical thinking source book for Malawi.* Domasi: MIE

Further reading

Everyday use of percentages: *www.staff.vu.edu.au/mcaonline/units/percent/pereve.html*

Elder, L (2002). *Teachers' manual for the miniature guide to critical thinking children.* The foundation for critical thinking Stand Up. http://*www.standup.org*

Crawford, A et al. (2005). *The teaching and strategies for the thinking classroom*. New York: The International Debate Association.

TOPIC 4 Teaching of taxes and premiums

Time 3 hours

Introduction

Government has responsibility of providing services such as education, health and potable water to its citizens. The government needs money to carry out these duties and responsibilities. Tax is one of the government sources of money.

In everyday life some goods are expensive and difficult to replace when they are damaged or lost. The same applies to accidents or sicknesses. In view of misfortune, people pay money to the insurance companies in order to be compensated. This money is what is termed as premium.

Studies have shown that some people do not understand the importance of taxes and premiums. Learners should be taught the concepts of taxes and premiums so that they should appreciate the importance of taxes and premiums and become responsible citizens.

In this topic, you will acquire appropriate teaching, learning and assessment skills for teaching taxes and premiums.

Success criteria

By the end of this topic, you must be able to:
- analyse how learners develop the concepts of taxes and premiums
- apply appropriate methodologies when teaching of taxes and premiums.
- use appropriate assessment methodologies in the teaching of taxes and premium

Background information

Tax is a compulsory contribution in form of money made by citizens to the government towards its expenditure. There are several types of taxes that people and organizations pay. These taxes are income tax, value added tax (VAT), withholding tax and exercise tax. Income tax is tax calculated based on an individual's income and it is also known as pay as you earn (PAYE). VAT is tax paid on most goods bought from shops. Withholding tax is tax one pays periodically for running business, and exercise tax is tax one pays on imported goods. These taxes are important because they enable the government to raise revenue, which is used for providing people with essential services, such as road construction and provision of health services.

Property, such as houses and household goods, may be lost due to damage or theft. The same applies to life due to sicknesses or injuries during accidents. People pay money to insurance companies in order to secure financial protection against such eventualities. When this happens, the property or life is said to be insured.

There are several types of insurance policies such as property and life insurance. The amount of money that is paid to insurance companies is called premium and is paid according to the agreed period of time such as monthly or yearly.

The concepts of taxes and premiums are well understood by learners when teachers use appropriate teaching, learning and assessment methodologies that involve learners in the lessons. Teachers need to use a variety of activities and examples to assist learners to understand the concepts and minimize confusion.

Task 1 Developing concepts of taxes and premiums

The topic of taxes and premiums is introduced in later years of primary school. By the time the topic is introduced, learners already have some knowledge of taxes and premiums. Teachers have to take advantage of this knowledge to make learners develop a deeper understanding of the topic. In this task, you will discuss prior knowledge and importance of taxes and premiums.

Activity 1 Discussing learners' prior knowledge on and importance of taxes and premiums

1 Explore situations in everyday life that can help children learn about taxes and premiums.
2 Discuss how you could use learners' prior knowledge in teaching taxes and premiums.
2 Discuss the importance of teaching taxes and premiums to primary school learners.
3 Explain how taxes and premiums are applied in everyday life.
4 Share your work with the class for discussion.

Activity 2 Analysing primary school curriculum expectations for teaching taxes and premiums

Analyse primary school instructional materials such as syllabuses, teachers' guides, and learners' books and do the following:

1 identify the concepts and skills of taxes and premiums that are taught in primary school. Summarise your work in the table

Class	Concepts

2 determine expected knowledge and skills that learners would acquire after learning taxes and premiums.
3 report your findings to the class for discussion

Task 2 Appropriate methodologies in the teaching and learning of taxes and premiums

For all learners to participate actively during lessons, teachers need to modify activities on taxes and premiums to ensure that learners with diverse educational needs participate actively and understand the concepts. By varying teaching strategies and using appropriate resources, learners with special educational needs can be motivated to participate actively in the lesson activities. In this task, you will explore appropriate methods and resources for teaching and learning of taxes and premiums.

Activity 1 Exploring the teaching of taxes and premiums

1 Identify resources that you would use in the teaching and learning of taxes and premiums.
2 Discuss how you would use the resources to teach the following:
   - types of taxes
   - calculating taxes
   - types of insurance policies
   - calculating premiums
   - solving practical problems on taxes and premiums
3 Discuss how you would modify the activities to accommodate learners with diverse educational needs.
4 Present your ideas to the class.

Tip
Use current tax calculation rates.

Task 3 Appropriate assessment methodologies in the teaching and learning of taxes and premiums

Assessment is an integral part in the process of teaching and learning. It helps the teacher to check learners' achievement levels as well as errors and misconceptions. Therefore, it is important to use appropriate techniques when assessing learners on taxes and premiums.

Activity 1 Analysing learners' strategies, misconceptions and errors

1 Ask if there is any student who taught taxes and premiums during their teaching practice.
2 Ask the students to share learners' misconceptions and errors.
3 Discuss other misconceptions and errors on taxes and premiums.
4 Suggest possible causes of the misconceptions and errors.
5 Discuss effective instructional approaches that you could use to address the misconceptions and errors.
6 Share your work with the class for discussion

Activity 2   Assessing learners on taxes and premiums

1   Develop word problems which you can use to assess primary school learners on taxes and premiums
2   Discuss ways of assessing learners on taxes and premiums.
3   Discuss tools for assessing learners on taxes and premiums.
4   Present your ideas to the class for discussion.

Summary
People pay income tax to the government depending on the amount of money they receive. These taxes help the government to provide various services to its citizens.

Premium is the monthly or yearly contribution one pays to an insurance company. This is one way of protecting people's property and even lives against theft or accidents.

Reflection and assessment

1   Describe precisely the difference between taxes and premiums.
2   With relevant examples, describe value added tax and custom duty.
3   The following table shows the rates for calculating income tax.

Income per month		Rate
First	K30,000.00	0%
Next	K5000.00	15%
In excess of K35,000.00		30%

Use the table to answer the following questions:
a.   calculate the income tax payable by Ms Kamtedza whose monthly salary income is K80, 0000.00.
b.   discuss how you would teach learners to solve the problem.
4   Study the following table on premiums.

Age	Annual premium per MK100,000.00 at death or after			
	20 year	25 yeas	30 years	35 years
20	MK10,000.00	MK9500.00	MK9000.00	MK8000.00
25	MK15,000.00	MK14,500.00	MK14,000.00	MK13,000.00
30	MK20,000.00	MK19,500.00	MK19,000.00	MK18,000.00
35	MK2,500.00	MK2,450.00	MK2,400.00	MK2,300.00

Generate as many questions as possible.

Glossary

Premium monthly or yearly contribution one pays to an insurance company.

Tax compulsory contribution made by an individual or organization to the state towards its expenditure.

References

Malewezi, F., & Mwanjabe, B. (1989). *Secondary school mathematics book 1.* Blantyre: Dzuka.

Ministry of Education, Science and Technology (2009). *Mathematics teacher's guide for Standard 8.* Domasi: MIE.

Ministry of Education, Science and Technology (2013). *Malawi primary, secondary and teacher education critical thinking sourcebook for Malawi.* Domasi: MIE.

Further reading

Question and answer on the premium tax credit.

http: www.irs.gov/affordable-care.../questions-and-answers-on-the-premium-tax-credit

Crawford, A et al (2005). *The teaching and strategies for the thinking classroom.* New York: The International Debate Association

Elder, L. (2002). *Teachers' manual for the miniature guide to critical thinking for children.* The foundation for critical thinking Stand Up. *www.standup.org*

Time   4 hours

Introduction

Simple and compound interest is one of the topics that is taught in primary school. Most people borrow money whether in the form of student loan, car loan or home loan. Likewise, people save money in the bank. Financial management is very important as it helps to make informed decisions about what to do with the money. Therefore, learners have to learn simple and compound interest so that they are aware of how interests are calculated. In addition, simple and compound interest form a basis for further studies.

In this topic, you will analyse how learners develop knowledge of simple and compound interest. You will also explore appropriate methodologies for teaching and assessing learners on simple and compound interest. The knowledge and skills of this topic will help you teach and assess primary school learners effectively.

Success criteria

By the end of this topic, you must be able to:

- analyse how learners develop the concept of simple and compound interest
- apply appropriate methodologies in the teaching of simple and compound interest

- use appropriate assessment methodologies in the teaching of simple and compound interest

Background information

Most people borrow money whether in the form of student loan, car loan or home loan. In the same way, people save money in the bank. Financial management is very important as it helps make rational decisions about what to do with money.

When you deposit money into your savings account, it means that you are lending it to the bank. The bank may use the money to provide loans to other customers. The money borrowed from the bank or deposited into your savings account attracts interest. The interest charged or profit made on the principal depends on three factors: interest rate, length of time of the investment and type of interest. There are two types of interest that can be charged; simple interest and compound interest.

The original amount invested or borrowed is called principal or present value. The total amount that the lender is paid back is called the future value. The future value includes the original amount lent (principal) and the lender's profit or interest, That is, Future Value (FV) = Principal (P) + Interest. To calculate simple interest, multiply the principal by time and by rate. On the other hand, compound interest is calculated annually.

That is, find the interest for one year first. The amount for year one becomes principal for year two. Then, calculate interest for year two. This process continues up to the final year. Alternatively, compound amount or future value, can be calculated using the following formula:

$FV = P\left(1 + \frac{R}{100}\right)^n$, where FV is future value or amount, P is principal, R is rate, and n is time in years.

Simple and compound interests are well understood by learners when teachers use appropriate teaching, learning and assessment methodologies that involve learners in the lessons. Teachers need to use a variety of activities and examples to assist learners understand the topic.

Task 1   Analysing how learners develop the concepts of simple and compound interest

Learners come to school with some experiences of simple and compound interest. They might have acquired these experiences from their homes and communities through village banks. However, the experiences are informal but have an influence on how they learn this topic in school. In this task, you will analyse learners' prior knowledge on simple and compound interests and identify curriculum expectations on the topic.

Activity 1   Discussing learners' prior knowledge on and importance of simple and compound interest

1   Explore situations in everyday life that help children acquire knowledge of simple and compound interest.
2   Discuss how you would elicit learners' prior knowledge on simple and compound interest.
3   Discuss why interest is charged on the money that people borrow from money lending institutions like banks.
4   Discuss the importance of teaching and learning simple and compound interest to primary school learners.
5   Explain how knowledge of simple and compound interest is used in everyday life.
6   Report your ideas to the class for discussion.

Activity 2   Analysing primary school curriculum expectations for teaching simple and compound interest

Analyse primary school instructional materials such as syllabuses, teachers' guides, and learners' books and other relevant sources and carry out the following:
1   identify concepts and skills of simple and compound interest that are taught in primary school. Present your work in a table

Class	Concepts

2   determine the expected knowledge and skills that learners would acquire after learning simple and compound interest.

3   share your findings with the class for discussion

Task 2   Appropriate methodologies in teaching and learning of simple and compound interest

Teaching simple and compound interest to primary school learners requires use of appropriate teaching and learning methodologies. Many teachers approach simple and compound interest differently depending on their content and pedagogical knowledge (PCK). Teachers need to understand the instructional materials and skills expected at each level in the primary school curriculum. In this task, you will explore appropriate teaching and learning methodologies for teaching simple and compound interest.

Activity 1   Exploring the teaching and learning of simple interest

1   Discuss how you would teach the following on simple interest:
    - deriving formula for simple interest
    - solving problems on simple interest

2   Present your ideas to the class for discussion.

Activity 2   Exploring the teaching and learning of compound interest

1   Discuss how you would teach solving problems on compound interest.

2   Present your ideas to the whole class for discussion.

> **Tip**
> Modify the activities to accommodate learners with diverse educational needs.

Activity 3   Analysing learners' strategies, misconceptions and errors on simple and compound interest

1   Analyse learners' strategies on simple and compound interest.

2   Identify learners' misconceptions and errors.

3   Explore possible sources and solutions to the misconceptions and errors

4   Present your ideas in plenary

> **Tips**
> - You may use learners' work on simple and compound interest from demonstration or nearby primary school.
> - You may also need to observe or watch a video lesson on simple and compound interest.
> - Research in other sources such as online articles and library books to understand learners' misconceptions and errors on simple and compound interest.

Task 3   Appropriate methodologies in the teaching and learning of simple and compound interest.

Assessment is an integral part of teaching and learning process. It informs how well teaching and learning progress. In this task, you will explore appropriate assessment methodologies in the teaching and learning of simple and compound interest. This will help you get necessary feedback from your learners and eventually use appropriate methodologies to adequately support them.

Activity 1   Discussing ways of assessing learners on simple and compound interest

1   Discuss ways of assessing learners on simple and compound interest.
2   Develop questions that you would use to assess learners on simple and compound interest.
3   Present your work to the class for discussion.

Activity 2   Developing assessment tools for assessing learners on simple and compound interest

1   Prepare a lesson plan on any sub concept of simple and compound interest.
2   Develop a checklist that you would use to assess learners' competencies.
3   Evaluate how the checklist has been used.

Reflection and assessment

1   a. Find the time in which K65, 000.00 will earn K164, 000.00 simple interest at 6% per annum.
   i.   Explore strategies that you would use to teach learners how to solve the question.
   ii. Explain how each strategy would be used to solve the question.
   b. Calculate compound interest on K10, 000.00 for 2 years at 20% per annum.
   i.   Explore strategies that you would use to teach learners how to solve the question.
   ii. Explain how each strategy would be used to solve the question.

2   A business lady borrows K300, 000, 000.00 to buy a car. She agrees to pay the money back in 2 years, paying simple interest at $8\frac{1}{2}$% per annum.

   a. Calculate the amount that the lady is supposed to pay back.

3   a. Calculate the time taken for K120,800.00 to yield an interest of K22,400.00 at 5% per annum.
   b. At what rate will K287,200.00 yield an interest of K359,000.00 in 5 years?

4   Mayamiko visited her uncle on Christmas day and was given a gift of K60, 00.00. She decides to save her money in the bank. She is offered two savings' options.

Option A: 10.5% per annum (p.a.)
Option B: 10.5% per annum compounded yearly
Mayamiko was then presented with the following future value table to help her make up her mind:

Simple Interest		Compound interest	
End year	Amount (Kwacha)	End year	Amount (Kwacha)
0	60000	0	60000.00
1	66300	1	66300.00
2	72600	2	73261.50
3	78900	3	80953.96
4	85200	4	89454.12
5	91500	5	98846.81
….		…	
10	123000	10	162844.85

Using the table, generate questions that you would ask your learners.

Tips

There are several questions that you can ask your learners, such as;
- What option do you think Mayamiko would go for and why?
- What is the difference between simple interest and compound interest?
- Why does compound interest yield more after the first year?

5 Calculate the difference between the compound interest and simple interest on K12, 000.00 for 2 years at 10% per annum.

6 Find the sum in which K5, 500.00 will amount to in 2 years at 20% per annum compound interest.

7 At what rate will K16, 000.00 amount to K17, 979.20 for 2 years compound interest?

Summary
Simple interest and compound interest are calculated using formulae. The formula for simple interest is
I = P × T × R/100, where P is principal, R is rate and T is time for which money is borrowed or saved. For compound interest, the interest is added to the principal at the end of each year or interval. So, to find amount, the formula is A or $FV = P\left(1 + \frac{R}{100}\right)^n$, where A or FV is the amount or future value, P is principal, n is time and R is rate which is in percentage. The formula for compound interest is CI = A – P. Learners should be taught in such a way that it is relevant to their real life experiences.

Glossary
Amount	the sum of principal and interest
Annum	year
Interest	money paid as a benefit for use of another person's money
Principal	amount of money borrowed or saved
Rate	interest expressed as a percentage of the principal

References

Ministry of Education, Science and Technology (2005). *Initial primary teacher education teaching syllabus*. Domasi: MIE.

Ministry of Education, Science and Technology (2008). *Initial primary teacher education students' book*. Domasi: MIE.

Ministry of Education, Science and Technology (2008). *Mathematics teacher's guide for Standard 8*. Domasi: MIE.

Ministry of Education, Science and Technology (2013). *Malawi primary, secondary and teacher education critical thinking sourcebook for Malawi*. Domasi: MIE.

Further reading

Elder, L. (2002). *Teachers' manual for miniature guide to critical thinking for children. The foundation for critical thinking stand up http://www.standup.org*

Time 1 hour

Introduction

People and organisations keep financial records for various reasons. The records serve as a source of information, frame of reference for future projections, accountability, for decision making and to check whether they are operating at a profit or loss. Clubs and societies in many schools keep financial records for various reasons. Therefore, the knowledge of simple accounts is useful to learners so that they apply it in their everyday life.

In this topic, you will learn how learners develop knowledge of simple accounts. You will also learn appropriate teaching, learning and assessment methodologies on simple accounts.

Success criteria

By the end of this topic, you must be able to:
- analyse how learners develop the concept of simple accounts
- apply appropriate methodologies when teaching simple accounts
- use appropriate assessment methodologies in the teaching of simple accounts

Background information

Simple accounts involve the keeping of financial records in which debits and credits for a particular activity or transaction are entered. There are three types of simple accounts that are taught in primary school. These are cash account, bank account and cash book. Cash account deals with income and expenditure of cash in hand while bank account is a record of money received and payment made through the bank. Cash book, on the other hand, combines cash account and bank account. See Figure 6.1 showing cash account, bank account and cash book.

Dr   Cr

Date	Income	Amount		Date	Payment	Amount	
		K	T			K	T

Cash account

Dr   Cr

Date	Income	Amount		Date	Payment	Amount	
		K	T			K	T

Bank account

Dr   Cr

Date	Income	Cash		Bank		Date	Expenditure	Cash		Bank	
		K	T	K	T			K	T	K	T

Cash book

Fig 6.1: Cash account, bank account and cash book

Learners should be made aware that knowledge and skills of simple accounts is important as it is applied in everyday life. For example, keeping financial records enhances transparency and accountability, future planning, and determining whether profits or losses are being made in any business.

Sometimes, learners experience difficulties in learning simple accounts. They may also have challenges in making connections between records of income and those of expenditure which are contained in simple accounts. Therefore, it is important for teachers to use appropriate methodologies for the teaching and learning of simple accounts so that learners acquire appropriate knowledge and skills.

Task 1   Analysing how learners develop the concept of simple accounts

Learners come to school with some experiences of keeping cash records from various businesses that they may have or through farming businesses that their parents or guardians may carry. In this task, you will discuss learners' prior knowledge and analyse primary school curriculum expectations on simple accounts.

Activity 1   Discussing learners' prior knowledge on and importance of simple accounts

1   Explore situations in everyday life that help children acquire knowledge of simple accounts.
2   Explain how you could elicit learners' prior knowledge on simple accounts.
3   Discuss the importance of learning simple accounts for primary school learners.
4   Explain how knowledge of simple accounts is applied in everyday life.
5   Share your ideas in plenary.

Activity 2   Analysing primary school curriculum expectations for teaching simple accounts

Analyse primary school instructional materials such as syllabuses, teachers' guides, learners' books and other relevant sources and carry out the following:

1   identify the concepts and skills of simple accounts that are taught in primary school. Summarise your work in a table

Class	Concepts

2 determine the expected knowledge and skills that learners would acquire after learning simple accounts

3 share your findings with fellow students

Task 2 Appropriate methodologies in teaching and learning of simple accounts

Teaching simple accounts to primary school learners requires use of appropriate teaching and learning methodologies. The approaches that many teachers use differ from one teacher to another depending, among others, on their content and pedagogical knowledge (CK and PCK respectively). In this task, you will explore activities and appropriate teaching and learning methodologies for teaching simple accounts.

Activity Exploring the teaching and learning of simple accounts

1 Identify resources that you would use to teach simple accounts.

2 Discuss how to teach each of the following on simple accounts using the identified resources:
   - introducing and balancing cash account
   - introducing and balancing bank account
   - introducing and balancing cash book

3 Present your work to the class.

Tip
You can use spreadsheet to balance your accounts.

Task 3 Appropriate assessment methodologies in the teaching and learning of simple accounts

The importance of assessment in the teaching and learning process cannot be overemphasised. In this task, you will explore appropriate assessment methodologies in the teaching and learning of simple accounts. This will help you get necessary feedback from your learners and eventually use appropriate methodologies to effectively support them.

Activity Discussing appropriate ways of assessing learners on simple accounts

1 Prepare a lesson plan on any concept of simple accounts.

2 Prepare questions that you would use to assess learners on simple accounts.

3 Develop a checklist that you would use to assess learners' competencies on simple accounts.

4 Peer teach the lesson.

5 Evaluate the lesson focusing on ways of assessment and use of the checklist.

Summary

Simple accounts are records of income and expenditure made within a specific period of time. There are three types of simple accounts; namely cash account, bank account and cash book. Learners should be engaged in various learning activities so that they understand and appreciate the importance of keeping financial records.

Reflection and assessment

1 Develop a rubric that you would use to assess learners on simple accounts.

2 Using the following as an example, explain how you could teach cash account given the following transactions made by Bwenzi Club at Namitengo Primary School:

May 2, 2017	balance in cash	K15000.00
May 3, 2017	sold beans at	K12500.00
May 4, 2017	sold maize at	K8000.00
May 5, 2017	paid wages	K5000.00
May 6, 2017	sold tomatoes at	K7500.00
May 7, 2017	paid transport	K3000.00
May 8, 2017	subscription fee	K4000.00
May 10, 2017	bought fertilizer	K10000.00
May 11, 2017	bought seeds	K1200.00

3 Using the following as an example, explain how you could teach bank account using the following transactions made by Mr Thokoloshe:

June 1, 2018   balance in bank K370, 000.00
June 4, 2018   bought bicycle by cheque   K65, 500.00
June 5, 2018   paid house rent by cheque K80, 000.00
June 12, 2018   deposited cash   K425, 000.00
June 20, 2018   deposited a cheque   K120, 000.00
June 28, 2018   deposited a cheque   K512, 000.00
June 30, 2018   paid by cheque   K300 100.00

4 Using the following as an example, explain how you could teach a cash book using the following transactions made by Mrs Dziko:

October 1, 2016   balance in cash K500, 000.00
October 4, 2016   bought goods with cash K60, 000.00
October 8, 2016   paid wages in cash K200, 000.00
October 9, 2016   paid rent and rates in cash K60, 000.00
October 10, 2016   sold goods K360, 000.00 cash
October 11, 2016   bought goods for K120, 000.00 cash
October 15, 2016   paid K360, 000.00 in cash for debts
October 17, 2016   banked K200, 000.00 cash
October 22, 2016   withdrew K200, 000.00 for office use
October 25, 2016   withdrew K200, 000.00 for office use
October 26, 2016   bought by cheque goods worth K300, 000.00
October 27, 2016   sold goods worth K400, 000.00 cash
October 28, 2016   paid K100, 000.00 by cheque
October 29, 2016   sold goods in cash K720, 000.00
October 31, 2016   banked K430, 000.00

Glossary

Account   a record in which debits and credits for a particular activity or transaction are entered

Balance   the difference between the total debits and the total credits in an account.

Balance brought forward money or goods recorded from the previous account

Balance carried forward money or goods that are taken to the next account

Bank account record of revenue received and payment made in the bank

Cash account record of revenue received and payment made by cash.

Cash book   record which combines cash account and bank account

Debit   amount of money paid in

Payments   money or goods paid out

Credit   amount of money or goods paid out

References

Ministry of Education, Science and Technology (2008). *Mathematics learners' books for Standard 7*. Domasi: MIE.

Ministry of Education, Science and Technology (2008). *Mathematics teachers'guide for Standard 7*. Domasi: MIE

Ministry of Education, Science and Technology (2008). *Mathematics learners' books for Standard 8*. Domasi: MIE.

Ministry of Education, Science and Technology (2008). *Mathematics teachers'guide for Standard 8*. Domasi: MIE

Ministry of Education, Science and Technology (2013). *Malawi primary, secondary and teacher education critical thinking sourcebook for Malawi*. Domasi: MIE.

Further reading

Temple, C. (2001). *Reading and writing for critical thinking in higher education: Critical thinking across the curriculum*. New York: Open Society Institute/RWCT Project www.rwt.org

TOPIC 7 Teaching of patterns

Time 6 hours

Introduction
Patterns have a wide application in real life situations. For example, patterns are used in clothing, weaving, painting, pottery, dances and carpentry. Understanding patterns provides opportunities like predicting future events, discovering new things, and make generalisations. The knowledge of patterns will help learners understand other mathematical concepts such as arithmetic and geometric progressions. Good knowledge of patterns will also help learners make informed career choices such as brick laying, designing and tailoring.

In this topic, you will analyse how learners develop knowledge of patterns and apply appropriate methodologies for teaching, learning and assessing learners on patterns.

Success criteria
By the end of the topic, you must be able to:
- analyse how learners develop the concept of patterns
- apply appropriate methodologies in the teaching and learning of patterns
- use appropriate assessment methodologies in the teaching and learning of patterns

Background information
A pattern is a systematic arrangement of numbers, shapes or other elements according to a defined rule. Patterns are found in objects, games, songs, clothes and the environment in general. Learners interact with patterns in their everyday undertakings.

The national primary curriculum offers three types of patterns which are simple, number and geometric. Simple patterns are made by arranging or drawing objects in a systematic way. These objects are the ones learners interact with in their everyday situations like stones, shapes, leaves, pencils and desks. For example, extend the pattern that follows with the next two shapes.

□◇□◇ ____ ____ ____

Extending simple patterns implies adding more objects to the previous objects based on the rule implied in the pattern. The rules of patterns are procedures that are used to generate and extend patterns. For example, in the previous pattern, to find the next shape, the rule is 'a rectangle followed by a rombus'. Rules are generated from an existing pattern or can be developed so that patterns can be generated.

A number pattern uses an arrangement of numbers according to a given rule. Learners count numbers in a pattern

knowingly or unknowingly. For example, 121, 111, 101, 91, 81, 71… in which the rule is "subtract 10 from the previous number to get the next number". Number patterns are extended in the same way as simple patterns.

A geometric pattern is an arrangement of objects or shapes that forms geometric shapes like triangle, rectangle and square. A set of the number of objects counted in each position of the pattern forms a geometric number pattern. For example, in a triangular pattern, a set of the number of objects in each position forms a triangular number pattern.

or
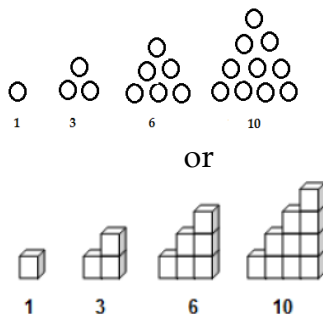
Simple patterns are well understood by learners when teachers use appropriate teaching, learning and assessment methodologies that involve learners in the lessons.

Task 1 Developing the concepts of patterns

Patterns are all around us. The arrangements of window panes in our classes and homes have patterns in them. Arrangements of pictures and other graphics on fabrics also have patterns in them. In order to teach patterns effectively, knowledge should develop

from what learners already know to what they do not know. An understanding of learners' prior knowledge on patterns will help you use appropriate strategies and resources to teach the topic effectively.

Activity 1 Establishing learners' prior knowledge and importance of patterns

1 Explore situations in everyday life that may help children learn about patterns.
2 Discuss the importance of patterns in relation to learners' everyday life.
3 Discuss how knowledge of patterns is applied in everyday life.
4 Share your work with the class.

Activity 2 Discussing primary school curriculum expectations on simple patterns

Analyse primary school curriculum materials such as syllabuses, teachers' guides and learners' books and do the following:

1 identify the concepts and skills of simple patterns that are taught in primary school. Present your work in a table

Class	Concepts

2 determine the expected knowledge and skills that learners would acquire after learning simple patterns.
3 report your work to the class for discussion

Task 2 Applying appropriate methodologies in the teaching and learning of patterns

Teaching and learning of patterns require use of appropriate teaching and learning methodologies. In this task, you will explore appropriate teaching and learning methodologies for teaching simple, number and geometric patterns.

Activity 1 Exploring the teaching and learning of simple, number and geometric patterns

1 Identify resources that you would use to teach simple patterns.
2 Discuss how you would use the resources identified to teach each of the following on simple patterns:
    - identifying patterns in the environment
    - establishing rules for patterns
    - completing patterns
    - extending patterns
    - generating patterns
3 Discuss how you would modify the activities to accommodate learners with diverse educational needs.
4 Report your ideas to the whole class.

Activity 2 Using technology in the teaching and learning of patterns

1 Design different types of patterns using computer application softwares like Microsoft word, Paint software and Geo Gebra software.
2 Print out the work and present it to the class.

Task 3 Using appropriate methodologies in the teaching and learning of patterns

Assessment is an essential component in the teaching and learning process. It feeds both the teacher and learners with information on the successes and challenges of the teaching and learning process. In this task, you will explore appropriate assessment methodologies in the teaching and learning of patterns. This will help you to apply appropriate methodologies to support learners.

Activity 2 Analysing learners' strategies, misconceptions and errors on patterns

1 Analyse learners' strategies on simple patterns.
2 Identify learners' misconceptions and errors on simple patterns
3 Explore possible sources and solutions to the problems.
4 Present your work to the class for discussion.

Tips
- You may need to use learners' work on patterns.
- You may also require to observe a lesson or watch a video lesson on patterns and use your teaching practice experiences on patterns.
- Research more information on misconceptions and errors from other sources like books and online articles.

Activity 2 Exploring appropriate assessment methodologies in the teaching and learning of patterns

1 Prepare a lesson plan on any concept of patterns and clearly indicate assessment strategies you will use.
2 Micro-teach the lesson.
3 Evaluate the lesson by focusing on assessment strategies you will have used.

Summary

A pattern is a systematic arrangement of numbers, shapes or other elements according to a specified rule. There are three types of patterns taught in the primary school namely; simple pattern, number pattern and geometric pattern. Patterns are extended based on the identified rule that is used to generate the pattern.

Teaching and learning of patterns require application of appropriate teaching, learning and assessment methodologies. Knowledge and skills from patterns will help learners understand other topics in mathematics and appreciate their environment.

Reflection and assessment

1 List resources that can be used for the teaching and learning of number patterns.
2 Using fanta, coca-cola, sprite and cocopina bottle tops, make as many patterns as possible and display them in class.

3 a. Extend each of the following simple patterns to three terms.
   i. XO XXOO XXXOOO

   _____ _____ _____
   ii. / \ / \ / _____ _____

   _____
   iii.

   _____ _____

   _____
   b. Explain how you can teach learners to extend each of the patterns in 3a.
4 Generate as many patterns as possible on each type of pattern that can be used in a lesson as an example and an exercise.
5 At Chiyo primary school, Mrs. Ntayanyemba and her learners planted cabbage in the school vegetable garden. She planted 2 plants in the first section, 4 plants in the second section and 6 plants in the third section. Explain how you would teach learners to:
   a. create this pattern using blocks
   b. find the number of plants there will be in the tenth section if the pattern continues in the same fashion.
6 The shapes below are triangular tables arranged in a restaurant in positions 1, 2 and 3.

   1   2   3
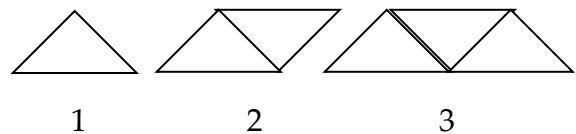
a. Explain how you would teach learners to:
   i. find the number of chairs for tables 1, 2 and 3 if there is one chair on each side of a table.
   ii. calculate how many chairs there will be on the 10thtable.
b. Prepare a marking key for the question.
c. Suggest learners' possible misconceptions and errors on the solution.

Glossary

Pattern a systematic arrangement of numbers, shapes or other elements according to a specified rule

Geometric pattern arrangement of objects or shapes in a pattern which forms geometric shapes

Rectangular numbers a set of number of objects or shapes which can be arranged in rows and columns to form rectangular shapes

Square numbers a set of numbers of objects or shapes which can be used to form square shapes

Triangular numbers a set of numbers of objects or shapes which can be used to form triangular shapes

Reference

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 1.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 2.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 3.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 4.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 5.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 6.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 7.* Domasi: MIE.

Malawi Institute of Education (2009). *Malawi primary education mathematics learners book for Standard 8.* Domasi: MIE.

Ministry of Education, Science and Technology (2010). *Initial primary teacher education through open and distance learning (ODL). Numeracy and mathematics. Module 3.* Lilongwe. DTED.

Malawi Institute of Education (2008). *Initial primary teacher education numeracy and mathematics. Lecturer's handbook*. Domasi: MIE.

Malawi Institute of Education (2008). *Initial primary teacher education numeracy and mathematics. Students' handbook*. Domasi: MIE.

Further reading

https://mathteachingstrategies.wordpress.com/2009/04/02/big-idea-patterns/

TOPIC 8   Teaching of algebraic expressions

Time 4 hours

Introduction

In some situations, letters of the alphabet are used to represent numbers. In mathematics, a statement which consists of numbers, variables and basic operations is called an algebraic expression. An understanding of algebraic expressions assists learners to go beyond the use of arithmetic operations. Children learn to use symbols to represent numbers and express mathematical relationships. Generally, using variables helps to simplify mathematical problems. An understanding of algebraic expressions enables learners to develop abstract reasoning skills which is necessary in other related subjects such as science. For this reason, teachers need to apply effective teaching and learning strategies to help learners develop reading, writing and computing skills of algebraic expressions.

In this topic, you will explore methodologies and assessment techniques in the teaching and learning of algebraic expressions in primary school. You will also analyse learners' misconceptions and errors on algebraic expressions and suggest strategies to address them.

Success criteria

By the end of this topic, you must be able to:

- analyse how learners develop the concept of algebraic expressions
- apply appropriate methodologies when teaching algebraic expressions
- use appropriate assessment methodologies in the teaching and learning of algebraic expressions

Background information

An algebraic expression is a mathematical sentence that contains numbers, variables and basic operations, for example, $5x + 4y$. Algebraic expressions also comprise coefficients which show how many times a particular variable has been added. For example, in the expression $3x + 2$, the coefficient of x is 3 while 2 is a constant term or simply a number. The operations in an algebraic expression act like a set of instructions that indicate what to do with the numbers and variables.

In any algebraic expression, the parts that are connected by addition or subtraction signs are called terms. For example, the expression $5x - 4$ has two terms, that is, $5x$ and -4. The terms which have same variables that are raised to the same power are called like terms while those with different variables are called unlike terms.

Algebraic expressions have different names according to the number of terms

that they have. For example, a monomial consists of one term, such as 4k. A binomial expression has two terms, like 3x – 5 and a trinomial expression consists of three terms, for instance $10a^2 + 3a – 1$. The value of an algebraic expression depends on the individual values of the variables. Such a value can be found by substituting the values of the variables in the expression and simplifying it. To simplify an algebraic expression is to write it in the most compact form without changing its value. This involves collecting like terms and then, adding and subtracting them to form a single term. The terms can also be simplified by multiplying and dividing them.

Algebraic expressions can be expanded so that all the operations are expressed in full. This involves multiplying each term inside the brackets by the number or variable outside the brackets. For example, in the expression 3(m + 7) both *m* and *7* are multiplied by 3 to get 3m + 21. The expression such as (a + b)(c + d) can be expanded using the distributive law. This means that each term in the first bracket is multiplied by each term in the second bracket as follows:
(a + b)(c + d) = (a + b)c + (a + b)d = c(a + b) + d(a + b) = ac + bc + ad + bd.

The opposite process of expanding algebraic expressions is known as factorising. Factoisation involves turning the sum of terms into a product of factors. In order to do this, you have to determine the highest common factor (HCF) of the

given terms and write the expression in brackets. For example, $12x – 8$ can be factorised as $4(3x – 2)$. Expressions, such as $y^2 – 4y + 5y – 20$, can be factorised by grouping terms into pairs as follows: $y^2 + 5y – 4y – 20 = y(y - 4) + 5(y -4)$ which gives$(y + 5)(y - 4)$. The binomial where both terms are perfect squares and one of them is subtracting from the other can be factorised using the rule for difference of two squares, that is $a^2 – b^2 = (a – b)(a + b)$. Algebraic expressions with 2 as the highest power of the variable are known as quadratic expressions. A standard quadratic trinomial in x is written in the form ax^2 + bx + c, where a, b and c are constants and a $\neq$ 0. To factorise a quadratic expression, such as $2x^2 + x – 300$, you start by multiplying the first term by the last term (constant), that is $2x^2$ X $– 300$ to get $– 600x^2$. Then find factors of $600x^2$ that when added give the middle term (that is $25x$ and $– 24x$). Re-write the expression as $2x^2 + 25x – 24x – 300$. Finally, factorise the expression by grouping the terms as follows: $x(2x + 25) – 12(2x + 25)$ to get $(x – 12)(2x + 25)$.

Sometimes learners tend to develop their own meanings and explanations of concepts being taught to them. In some cases, their conceptions could be in conflict with the correct meanings of mathematical concepts. This results in misconceptions. These misconceptions may impede children's learning of

concepts if they are not corrected appropriately resulting in errors in the process of solving mathematical problems.

The following are some misconceptions and errors that learners may have on algebraic expressions:
- taking a variable as a label for an object. For example, in the expression 3b – 4, a learner could say that *b* stands for *books* or *bananas*
- thinking that a variable stands for a specific unknown and that different letters must represent different numbers. The learner may not understand that a variable can represent multiple values
- performing the operations of addition without considering the negative sign attached to the integer

The teaching and learning of algebraic expressions to primary school learners requires teachers to use appropriate methodologies so that learners understand concepts well. Teachers also need to use appropriate assessment techniques in order to adequately support learners.

Task 1 Analysing how learners develop knowledge of algebraic expressions

Algebraic expressions play an important role in our everyday situations. In this task, you will analyse learners' prior knowledge on algebraic expressions. You will also identify primary school curriculum expectations on algebraic expressions.

Activity 1 Discussing learners' prior knowledge and importance of algebraic expressions

1 Explore situations in everyday life that may help children develop knowledge of algebraic expressions.
2 Discuss the importance of algebraic expressions to primary school learners.
3 Explain how knowledge of algebraic expressions is applied in everyday life.
4 Report your ideas to the class for discussion.

Activity 2 Analysing primary school curriculum expectations for teaching algebraic expressions

Analyse primary school instructional materials such as syllabuses, teacher's guides, learners' books and other relevant sources, and then carry out the following:

1 identify the concepts and skills of algebraic expressions that are taught in primary school. Show your findings in a table.

Class	Concepts

2 determine the expected knowledge and skills that learners would acquire after learning algebraic expressions
3 present your work to the class for discussion

Task 2 Applying appropriate methodologies in the teaching and learning of algebraic expressions

Effective teaching and learning of algebraic expressions requires learners to carry out various activities using relevant methods and resources. Teachers need to modify activities to accommodate learners with diverse education needs. In this task, you will explore activities in the teaching and learning of algebraic expressions.

Activity 1 Exploring appropriate methodologies in the teaching and learning of algebraic expressions

1 Identify resources which you would use to teach algebraic expressions
2 Discuss how you would use the identified resources to teach each of the following on algebraic expressions:
   - introducing variables
   - identifying terms in algebraic expressions
   - basic operations on algebraic expressions
   - expanding algebraic expressions
   - factorising algebraic expressions
   - factorising quadratic expressions
3 Explain how you would modify the activities to meet learners with diverse educational needs.
4 Share your ideas in plenary.

Activity 2 Analysing learners' strategies, misconceptions and errors on algebraic expressions

1 Analyse learners' strategies on algebraic expressions.
2 Identify learners' misconceptions and errors on algebraic expressions.
3 Suggest possible causes of the misconceptions and errors.
4 Discuss effective instructional approaches that you would use to address these misconceptions and errors.
5 Report your findings to the class for discussion.

Tips
- You may use learners' work on algebraic expressions.
- You may also reflect on your teaching practice experiences on algebraic expressions.
- Research from other sources to understand learners' misconceptions and errors on algebraic expressions.

Task 3 Appropriate assessment methodologies in teaching and learning of algebraic expressions

Assessment complements teaching and learning process. In this task, you will explore suitable ways of assessing learners on algebraic expressions. This will help you get necessary feedback from your learners and eventually use appropriate methodologies to support them.

Activity 1   Discussing ways of assessing learners on algebraic expressions

1   Prepare a lesson plan on how you could teach any one concept on algebraic expressions.
2   Peer teach the lesson.
3   Evaluate the lesson by focusing on ways of assessing learners.

Summary

An algebraic expression is a mathematical sentence that contains numbers, variables and basic operations. Algebraic expressions could be monomial, binomial and trinomial. The value of each algebraic expression depends on the values of the individual variables. An algebraic expression can be simplified by writing it in the most compact form without changing its value.

Expanding an algebraic expression involves writing it in full by multiplying each term inside the brackets by the number or variable outside the brackets. The opposite process of expanding an algebraic expression is known as factorizing. Factorisation changes a given sum of terms into a product of factors.

Reflection and assessment

1   What could be the pre-requisite knowledge for teaching algebraic expressions?
2   Discuss suitable methods and resources that you would use in the teaching and learning of algebraic expressions.

3   Develop a checklist that you could use to assess learners after teaching simplification of like and unlike terms.
4   a   Suggest problems that learners may experience when learning algebraic expressions.
   b   Explain how you would address the problems.

Glossary

Algebraic expression a collection of algebraic terms that are connected together by addition or subtraction signs

Coefficient   a number indicating how many times a variable is added

Factorise   to express as a product of factors

Like terms   the terms in an algebraic expression with same variable raised to the same power

Term   part of an algebraic expression connected together by an addition or a subtraction sign

Unlike terms   terms in an algebraic expression which have different variables or same variable but whose powers are different

Variable   a letter of the alphabet, like x or y, that can assume any value

References

Department of Teacher Education and Development (2010). *Initial primary teacher education through open and distance learning (ODL). Numeracy and mathematics. Module 2*. Lilongwe: DTED.

Lucariello, J., Tine, M., & Ganley, C. (2014). A formative assessment of students' algebraic variable misconceptions. *Journal of Mathematical Behaviour, 33*, 30-41.

Malawi Institute of Education (2008). *Initial primary teacher education. Numeracy and mathematics. Lecturers' book.* Domasi: MIE.

Malawi Institute of Education (2008). *Initial primary teacher education. Numeracy and mathematics. Students' book.* Domasi: MIE.

Thomo, F., Mugo, K., Maina, L., & Ondera, J. (2012*). Excel & succeed. Junior secondary mathematics. Form 1*. Nairobi: Longhorn Publishers (K) Ltd.

Further reading

Booth, J., Barbieri, C., Eyer, F., & Paré-Blagoev, E. (2014). Persistent and pernicious errors in algebraic problem solving. *Journal of Problem Solving, 7*(1), 3.

Mulungye, M., O'Connor, M., & Ndethiu, S. (2016). Sources of student errors and misconceptions in algebra and effectiveness of classroom practice remediation in Machakos County-Kenya. *Journal of Education and Practice, 7*(10), 31-33.

TOPIC 9   Teaching of equations

Time 6 hours

Introduction

An equation is an important mathematical concept that children learn in later years of primary school. It is practised in real life situations such as in business. Equation is analogous to a balance scale. For example, when two equal masses, such as a 1 kilogram stone and maize grains are placed onto two pans of a balance scale and they balance, this shows that the two quantities are equal.

Although the idea of equation is practised in real life, some learners may find it difficult to understand it. Therefore, teachers need to use a variety of methods and resources to promote active participation of all learners. In this way, learners would be able to develop relevant knowledge and skills which they can apply in their everyday life. An in-depth understanding of equations sets learners up for success in many areas of mathematics and other disciplines that require use of mathematical formulae.

In this topic, you will learn about various activities that are carried out in the teaching and learning of equations in primary school. You will also learn about misconceptions that learners may have and errors that they may make and then, suggest possible ways of addressing them.

Success criteria

By the end of this topic, you must be able to:

- analyse how learners develop concepts of equations
- apply appropriate methodologies in the teaching and learning of equations
- use appropriate techniques to assess learners on equations

Background information

Equations are statements of equality containing one or more variables. They are classified according to the highest power of the variable. A linear equation is an algebraic expression in which the highest power of the variable is 1. When graphed, the equation gives a straight line. Another type is quadratic equation which has 2 for the highest power of the variable. When it is graphed, a quadratic equation produces a curve which is referred to as a parabola.

Formula is an important concept which is commonly used in mathematics. This is an equation that is used to calculate mathematical concepts such as area and perimeter. The formula is usually written with one unknown expressed in terms of other unknowns. For example, the formula for the area of a rectangle is Length × Width, which may be shortened as $A = L \times W$. In this case, A is the subject of the formula. The formula can be rearranged to make any one of the unknowns the subject of the formula.

Solving an equation involves determining the values of the variables that fulfil the condition stated by the equation. The following steps can be followed when solving problems involving linear equations:

Step 1 : Define the problem: Read the given question and understand it.

Step 2 : Assign variables: Choose a letter (if it is not provided) to represent the unknown number.

Step 3 : Translate it into an equation: Formulate an equation.

Step 4 : Solve the equation: Find the value of the unknown.

Step 5 : Check the answer: Substitute the answer in the equation and check to see if it makes sense within the context of the problem.

In primary school, learners are taught linear equations in later years where they learn to form simple equations from given statements and solve them. They use strategies such as the following to solve equations:

a) Inspection (trial and perfect)**:** the method works through trial and error

b) Algebraic process**:** This process is about doing the same to both sides of the equation. For example, the value of x in the equation $x - 4 = 2$ is found by adding 4 to both sides. This process removes -4 from the left side as follows: $-4 + 4$ or $4 - 4$ leaving x as the subject of the formula.

Successful solving of equations requires conceptual understanding of all the features in the problem, such as equal sign, variables, like terms and operation signs.

Effective teaching and learning of equations demands active participation of learners in a variety of activities using appropriate resources. This enables learners to understand equations and be able to solve them. However, some learners may have misconceptions and may make errors which could interfere with the learning of the concepts if they are not corrected appropriately. In this case, teachers need to use appropriate techniques to address the misconceptions and errors for learners to develop appropriate conceptual knowledge of equations.

Through regular assessment, teachers can improve the teaching and learning process. For instance, by administering continuous assessment such as questioning and short exercises, teachers would identify the needs of learners and modify the teaching process to address them.

Task 1   Analysing how learners develop the concept of equations

Equations are commonly used in everyday life. As a result, learners may have prior knowledge of equations which can greatly influence what they learn. In this task, you will explore learners' prior knowledge and importance of equations.

Activity 1   Discussing learners' prior knowledge and importance of equations

1 Explore situations in everyday life that help children learn about equations.
2 Discuss the importance of teaching equations to primary school learners.
3 Explain how the knowledge of equations is applied in everyday life
4 Report your findings to the class for discussion

Activity 2   Analysing primary school curriculum expectations on equations

Analyse primary school instructional materials such as syllabuses, teacher's guides, learners' books and other relevant sources and do the following:

1 identify concepts and skills of equations that are taught in primary school.
   Present your work in a table

Class	Concepts

2 determine the expected knowledge and skills that learners would acquire after learning equations
3 share your findings with the class for discussion

Task 2   Applying appropriate methodologies in the teaching and learning of equations

Primary school learners must be engaged in a number of activities using relevant resources for them to develop meaningful knowledge and skills of equations. In this task, you will explore appropriate methods and resources for the teaching and learning of equations.

Activity 1   Discussing the teaching of equations

1 Identify resources that you can use to teach equations.
2 Discuss how you would use the identified resources to teach each of the following concepts on equations:
   - changing subject of formulae
   - solving linear equations
   - generating linear equations from word problems
3 Explain how you would modify the activities to meet learners with diverse educational needs.
4 Present your ideas to the class.

Activity 1   Analysing learners' strategies, misconceptions and errors on equations

1 Analyse learners' strategies on equations.
2 Identify learners' misconceptions and errors on equations.
3 Discuss possible sources of the misconceptions and errors.
4 Discuss effective instructional approaches that you could use to address the misconceptions and errors.
5 Report your findings to the class for discussion

Task 3 Using appropriate techniques to assess learners on equations

Assessment complements the teaching and learning process. In this task, you will use appropriate techniques to assess learners on equations.

Activity 1 Exploring ways of assessing learners on equations

1. Discuss suitable ways of assessing learners on equations.
2. Explain challenges that you would face when assessing learners on equations.
3. Suggest possible solutions to the challenges.
4. Share your ideas with the class for discussion.

Tips
- Use learners' work to analyse learners' strategies, errors and misconceptions.
- You should also reflect on your teaching practice experiences.

Summary

An equation is a statement of equality containing one or more variables. A linear equation is an algebraic expression in which the highest power of the variable is 1. A formula is an equation which is used to calculate mathematical concepts such as area and perimeter.

Reflection and assessment

1 Identify the concepts and skills that are taught to primary school learners on equations. Arrange them in a logical teaching order.
2 Explain, in point, form how you would introduce equations in primary school.
3 Discuss suitable methods and resources that you could employ in the teaching and learning of equations.
4 Consider the equation, $x - 5 = 6$. Explain the steps that you would follow to teach learners how to solve the equation.
5 Prepare a lesson plan for teaching any concept of equation in a particular class in primary school.
6 What could be the pre-requisite knowledge for introducing equations to primary school learners?
7 Develop a checklist that you could use to assess learners after teaching equations.
8 Suggest problems that learners may experience when solving equations.
9 Explain the importance of teaching equations to primary school learners.
10 Generate questions on linear equations and explain strategies that you can use to teach learners how to solve them.

Glossary

Equation a mathematical statement which shows equality of quantities

Formula statement with an equal sign showing the relationship between two or more variables

Linear equation an algebraic expression with 1 as the highest power of the unknown

Variable a symbol like x or y that can assume any value

References

Chikwakwa, R., & Suffolk, J. (2000). *Malawi junior secondary mathematics. Students' book 2.* Blantyre: Macmillan Malawi Ltd.

Department of Teacher Education and Development (2010). *Initial primary teacher education through open and distance learning (ODL). Numeracy and mathematics. Module 2.* Lilongwe: DTED.

Malawi Institute of Education (2008). *Initial primary teacher education: Numeracy and mathematics lecturers' book.* Domasi: MIE.

Malawi Institute of Education (2008). *Initial primary teacher education. Numeracy and mathematics. Students' book.* Domasi: MIE.

Thomo, F., Mugo, K., Maina, L., & Ondera, J. (2012*). Excel & succeed. Junior secondary mathematics. Form 1.* Nairobi: Longhorn Publishers (K) Ltd.

Further reading

Haylock, D. (2010). *Mathematics explained for primary teachers* (4th ed.). London: SAGE Publication Ltd.

Hansen, A. (2014). *Children's errors in mathematics. Understanding common misconceptions in primary schools.* Exeter: Learning Matters Ltd.

TOPIC 10 Teaching of inequalities

Time 4 hours

Introduction
In real life situations, not all quantities are equal. For example, you find that the mass of one object is greater than that of another object or one learner is taller than the other. Each of these statements indicates a relation that holds between quantities that are not equal in value and magnitude. In mathematics, this relation is called inequality. The concept of inequality is extremely useful when sharing and comparing quantities. The concept plays a critical role in developing conceptual understanding of other mathematical fields such as algebra and geometry. Furthermore, inequalities are useful in fields such as catering, sports, surveying, agriculture and manufacturing.

In primary school, learners are introduced to inequality symbols and how to use them in mathematical sentences. In this case, teachers should assist learners to have a clear understanding of inequalities in preparation for further mathematical concepts and be able to apply the knowledge and skills in everyday life.

In this topic, you will explore various activities and learn about various methodologies and assessment techniques in the teaching and learning of inequalities in primary school. You will also learn about some misconceptions and errors that learners may have and suggest possible techniques of addressing them.

Success criteria
By the end of this topic, you must be able to:
- analyse how learners develop the concept of inequalities
- use appropriate methodologies when teaching inequalities
- apply appropriate assessment methodologies in the teaching of inequalities

Background information
In mathematics, inequality is the relation between two quantities that are not equal. This concept is crucial for learners to understand other fields in mathematics such as algebra and geometry. An inequality is expressed in form of a mathematical sentence in which one mathematical expression is not equal to the other. Just like equations, letters are used in inequalities as notations for the unknown values. However, an inequality does not have an 'equals' sign, but uses the following inequality symbols: $<$, $>$, $\geq$ and $\leq$ with the following meanings:
- $>$ means 'is greater than', for example, *3 > 1*
- $<$ means 'is less than', for example, *2 < 5*
- $\geq$ means 'is greater than or equal to', for example, *speed is $\geq$ 80 km/h*
- $\leq$ means 'is less than or equal to', for example mass $\leq$ *10 kg*

The statement $x > 1$ means that any value greater than 1 is true for the inequality. Graphically, this can be represented on a number line as follows:
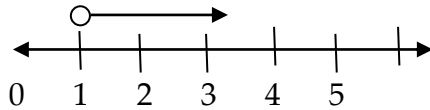

The starting point of an arrow for inequalities with the symbol < or > has an open point or circle (o) to indicate that the starting point, for this example 1, is not an element of the solution set. For our example, the integer solution set is {2, 3, 4, 5, …}. The dots denote that the values go to infinity (that is, the values continue forever in the same fashion).

While the inequality $x \geq 1$ can be represented graphically as follows:
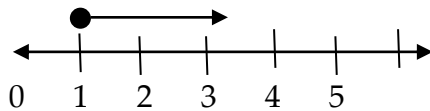
The starting point of the arrow for inequalities with the symbols ≤ or ≥ has a solid dot or closed circle (●). This indicates that the starting point, in this example, 1, is an element of the solution set. In the inequality $x \leq 3$, the value of x must be less than or equal to 3 and the integer solution set is {3, 2, 1, 0, – 1, ...}.

Sometimes an inequality contains two symbols such as $-1 < x \leq 3$. This means that x is a set of values that are greater than –1 (or –1 is less than x) and less than or equal to 3. This can be represented on a number line as follows:
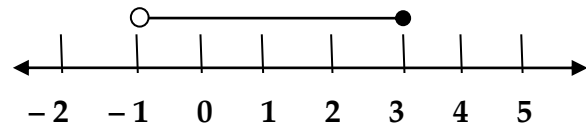
A linear inequality looks exactly like a linear equation with the inequality sign replacing the equality sign. To solve an inequality is to find numbers which can be substituted for the variable to make the inequality a true statement. Usually, the solution is a range of all values of the variable that fit the inequality mathematical statement or satisfies the inequality. The following rules must be followed when performing algebraic operations on inequalities:

Rule 1   Adding or subtracting the same quantity from both sides of an inequality leaves the inequality symbol unchanged.

Rule 2   Multiplying or dividing both sides by a positive number

leaves the inequality symbol unchanged.

Rule 3   Multiplying or dividing both sides by a negative number reverses the inequality symbol. For example, the symbol < changes to > and vice versa. For example, $-2x > 4$ can be simplified to get $x < -2$. (Show why this rule always works).

Tip

- It is important to establish why each of these rules always work.

Inequalities are introduced in later years of primary school. Although solving linear inequalities has some similarities to solving linear equations there are some few differences which learners may overlook and assume that solving linear inequalities requires similar processes. Hence, it is important for teachers to explicitly draw learners' attention to the differences between the two concepts. In addition, teachers should use appropriate methods and resources for learners to apply appropriate strategies to solve problems involving inequalities correctly.

In the process of teaching and learning of inequalities, some learners may have some misconceptions and make errors which may interfere with the learning of the concept. As a teacher, you need to use appropriate techniques to address various misconceptions and errors that learners may have. You should use a variety of sets of objects for modelling inequalities before introducing the symbols for learners to understand the concept. Through regular assessment using tools such as rubric and checklist, teachers could be able to identify the needs of learners and modify the teaching and learning process in order to teach them accordingly.

Task 1   Analysing how learners develop the concept of inequalities

Inequalities are commonly used in everyday life and figuring out how to interpret its language is an important step towards learning how to solve them in everyday contexts. In this task, you will elicit learners' prior knowledge and determine curriculum expectations on inequalities.

Activity 1   Discussing learners' prior knowledge and importance of inequalities

1   Explore situations in everyday life that may help children learn about inequalities.
2   Discuss the importance of teaching inequalities to primary school learners.
3   Explain how inequalities are applied in everyday life.
4   Report your ideas to the class for discussion.

Activity 2   Exploring primary school curriculum expectations on inequalities

Analyse primary school instructional materials such as syllabuses, teacher's

guides, learners' books and other relevant sources and carry out the following:

1 identify the concepts and skills of inequalities that are taught in primary school. Show your findings in a table.

Class	Concepts

2 determine expected knowledge and skills that learners would acquire after learning inequalities

3 present your work in plenary

Task 2 Appropriate methodologies in the teaching and learning of inequalities

The teaching and learning of inequalities require learners to carry out various activities using relevant resources for them to develop a clear understanding of the concept. In this task, you will explore appropriate methods and resources for the teaching and learning of inequalities.

Activity 1 Discussing the teaching of inequalities

1 Identify resources that you would use to teach inequalities.

2 Discuss how you would use the identified resources to teach each of the following concepts:
- inequality language and symbols
- rules governing inequalities
- solving problems on inequalities
- formulating word problems on inequalities

3 Explain how you would modify the activities to accommodate learners with diverse educational needs.

4 Present your ideas to the class for discussion.

Activity 2 Exploring learners' misconceptions and errors on inequalities

1 Analyse learners' strategies on inequalities.

2 Identify learners' misconceptions and errors on inequalities.

3 Discuss possible sources of the misconceptions and errors.

4 Discuss effective instructional approaches that you can use to address the misconceptions and errors.

5 Report your work to the class for discussion.

Task 3 Appropriate assessment methodologies in the teaching of inequalities

Assessment is an integral part of the teaching and learning process. Appropriate assessment methodologies inform how well teaching and learning progress. In this task, you will explore appropriate assessment methodologies in the teaching and learning of inequalities.

Activity 1 Discussing ways of assessing learners on inequalities

1 Discuss suitable ways of assessing learners on inequalities.

2 Explain challenges that you would face when assessing learners on inequalities.

3 Suggest possible solutions to the challenges.

4 Share your work with the class for discussion.

Activity 2   Developing tools for assessing learners on inequalities

1 Formulate word problems involving inequalities.
2 Develop a scoring rubric that you would use to assess learners on the word problems.
3 Suggest challenges that learners are likely to face when answering the questions (Hint: Challenges that learners may face in answering word problems).
4 Suggest ways that you could use to assist learners who face challenges in answering the word problems.
5 Report your ideas in plenary.

Summary

The relation between two quantities that are not equal is called inequality. This relation is expressed in form of a mathematical sentence in which one mathematical expression is not equal to the other. Unlike a linear equation, linear inequality uses the following symbols: $<$, $>$, $\geq$ and $\leq$. The inequality symbol $>$ means 'is greater than', $<$ means 'is less than', $\geq$ means 'is greater than or equal to' and $\leq$ means 'is less than or equal to'.

Graphically, inequalities can be represented on a number line. An open circle (o) at the starting point of an arrow means that the value at that point is not part of the solution set while a shaded circle (●) means the value at that point is part of the solution set.

Inequalities are introduced in later years of primary school. Learners carry out activities such as inserting inequality symbols in number sentences and solving simple inequalities.

In teaching and learning of inequalities, some learners could have some misconceptions and may make errors which may interfere with the learning of the concept. Teachers need to use appropriate techniques to address them.

Through regular assessment using appropriate tools such as rubric checklist, teachers could be able to identify the needs of learners and modify the teaching and learning process in order to assist them accordingly.

Reflection and assessment

1 What could be the pre-requisite knowledge for teaching inequalities to primary school learners?
2 Explain the importance of teaching inequalities to primary school learners.
3 Prepare a lesson plan for teaching any concept on inequalities in primary school.
4 James thought of a number x. He multiplied it by 3 and then subtracted 2 from the result. His answer was between 45 and 65. Explain the steps that a learner could follow to find the possible numbers that James could have thought of?

5   Suggest problems that learners may experience when learning inequalities.

Glossary

Inequality   a mathematical statement in which one quantity is greater than or less than the other

Infinity   without an end

Solving inequality   finding all values of the variable for which the inequality is true

References

Botty, H., Yusof, J., Shahrill, M., & Mahadi, M. (2015). Exploring students' understanding on 'inequalities'. *Mediterranean Journal of Social Sciences*, *6*(5), 218.

Department of Teacher Education and Development (2010). *Initial primary teacher education through open and distance learning (ODL). Numeracy and mathematics. Module 2.* Lilongwe: DTED.

Malawi Institute of Education (2008). *Initial Primary Teacher Education. Numeracy and mathematics. Lecturers' book.* Domasi: MIE.

Malawi Institute of Education (2008). *Initial primary teacher education. Numeracy and mathematics. Students' book.* Domasi: MIE.

Morrison, K., Slamang, M., & Greenstein, L. (2014). Study & master mathematics for Malawi. Form 2 students' book. Cape Town: Camridge University Press.

Further reading

El-khateeb, M. (2016). Errors analysis of solving linear inequalities among the preparatory year students at King Saud University. *Journal of Education and Practice*, *7*(12), 124-133.