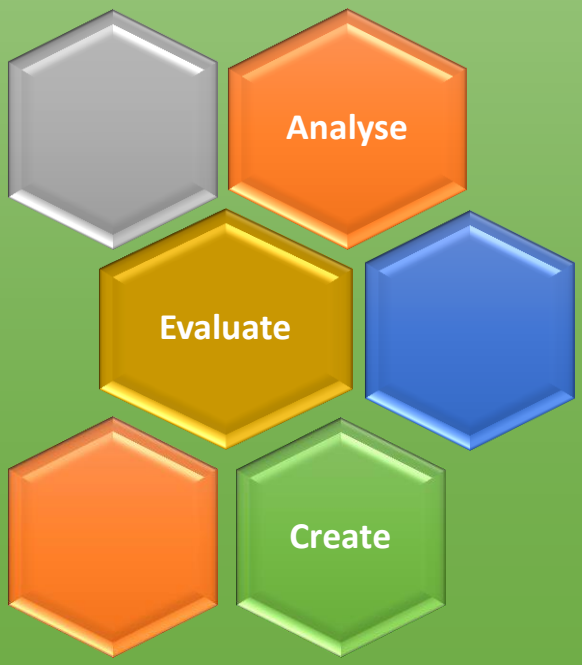
MALAWI PRIMARY AND TEACHER EDUCATION



CRITICAL THINKING SOURCEBOOK FOR

MATHEMATICS

Malawi Government

OPEN SOCIETY FOUNDATIONS

Malawi Institute of Education

Developed and published by

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P.O. Box 50

Domasi

Malawi

Email: miedirector@sdnp.org.mw

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PREFACE

In 2010, the Ministry of Education, Science and Technology (MoEST) and Malawi Institute of Education (MIE) embarked on activities to introduce critical thinking in schools across the country. Both institutions believe that critical thinking skills are essential for sound decision making by learners and citizens alike. In 2013, the *Critical Thinking Sourcebook for Malawi* (CTSM) was published, and it has been used by teachers and teacher educators to develop methods and strategies for the promotion of critical thinking in the classroom. The rationale for the initiative is to improve the quality of education in Malawi.

Through critical thinking, the Ministry of Education, Science and Technology expects that learners will be exposed to survival strategies and skills which will be applied to their daily lives, thereby improving their livelihoods as well as developing Malawi. The MoEST and MIE have taken the next step in the advancement of critical thinking in Malawi through the development of critical thinking sourcebooks in four of the key subject areas in primary schools: humanities, language and literacy, mathematics and sciences. The sourcebooks share some common sections and chapters, but they also contain subject-specific methods and strategies because critical thinking cannot be promoted in exactly the same way in every subject.

In the case of mathematics, for example, learners need to learn how to analyse problems that resemble the real-life purposes to which mathematics can be put and to solve such problems in the most rational way possible. In humanities, learners must develop skills to evaluate claims about healthy living in life skills and environmental conservation. In sciences, learners should be able to analyse natural phenomena and use scientific knowledge, skills, attitudes and values to solve everyday problems.

In languages and literacy classes, it is essential that learners cultivate their creativity through the reading and writing of poems and stories, and develop their ability to analyse information as they improve their listening and speaking skills. Although critical thinking skills can be developed in each of these areas, they are somewhat different by virtue of the subjects themselves. We believe these subject-specific sourcebooks will make an important contribution to the advancement of quality education in the country.

William Susuwele-Banda, PhD
Executive Director, Malawi Institute of Education

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Ms V. Manda	OSISA, South Africa
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Ms A. Nagoma	Maryam TTC
Ms E. Ndala	Maryam TTC
Mr S. Malukula	Maryam TTC
Mr M. Konyani	St. Joseph's TTC
Mr M. Sathula	St. Joseph's TTC
Mr J. Kampango	St. Joseph's TTC
Mr G. Bwanaluli	St. Joseph's TTC
Mr W. Mataka	Liwonde CDSS
Mr A. Nyasulu	Loudon TTC
Mr B. Mwafulirwa	Loudon TTC
Mr A. Phiri	Kasungu TTC
Mr A. Disi	Kasungu TTC
Ms B. Mambiya	Phalombe TTC
Mr B. Chikoti-Phiri	Machinga TTC
Ms R. Nyirenda	Machinga TTC
Ms M. Potani	Machinga TTC
Mr D. Nangwale	Chiradzulu TTC
Mr J. Kamzati	Chiradzulu TTC
Mr D. Tembo	DAPP Chilangoma TTC
Mr J. Bwenzi	DAPP Amalika TTC
Mr R. Malizeni	DAPP Amalika TTC
Mr B. Masangalawe	DAPP Amalika TTC
Ms A. Mzumara	Karonga TTC
Mr W. Lembe	Karonga TTC
Mr J. Mwangomba	Karonga TTC

Mr W. Nkhoma	Domasi College of Education
Ms C. Kumwamba	Domasi College of Education
Mr S. Kathyole	Emmanuel TTC
Mr K. Sunduzwayo Banda	DAPP Dowa TTC
Mr D. Mbingwa	DAPP Dowa TTC
Mr P. Namphande	Chancellor College
Dr S. Chiziwa	Chancellor College
Dr E. Kunkwenzu	Chancellor College
Mr P. Kapito	Chancellor College
Mr A. Walata	Blantyre TTC
Ms R. Makanga	Blantyre TTC
Ms R. Kuntaja	Blantyre TTC
Ms E. Lemani	Malawi Institute of Education
Ms E. Chinguwo	Malawi Institute of Education
Dr E. Kachisa	Malawi Institute of Education
Mr E. GhobedeNtonga	Malawi Institute of Education
Mr D. Kaambankadzanja	Malawi Institute of Education
Mr A. Kalambo	Malawi Institute of Education
Dr G. Chirwa	Malawi Institute of Education
Ms M. Katundulu	Malawi Institute of Education
Mr G. Makanani	Lilongwe TTC
Mr F. Macheke	Lilongwe TTC
Ms C. Soko	DIAS
Ms M. Chirwa	DTED
Mr J. Kholowa	Phalombe TTC

PRODUCTION TEAM FOR THIS BOOK

Editing	: Dr William Susuwele-Banda
Typesetting	: Mazganga Lino
Layout and Design	: Davie Kaambankadzanja

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CHAPTER ONE

Understanding Critical Thinking

Introduction

The term *critical thinking* combines the concepts of clarity and rationality in our reasoning with judgement, discernment and critique in our intellectual endeavors. Although thinking is an activity in which we are engaged during most of our waking hours, *critical thinking* indicates a more active process by which we evaluate knowledge and produce it ourselves. This chapter introduces the meaning and features of critical thinking that will be used throughout the sourcebook. The chapter further discusses the relationship between critical thinking and constructivism as well as the relationship between critical thinking and active learning.

Meaning and features of critical thinking

There are many definitions of critical thinking. For the purposes of the subject-specific sourcebooks, we will use the definition in the Critical Thinking Sourcebook for Malawi (CTSM). It states that critical thinking is "thinking that aims at reaching a well-founded judgment and hence, utilizes appropriate evaluative standards in an attempt to determine the true worth or merit of information" (CTSM,2013; p. 1). In other words, critical thinking is the process by which we make defensible decisions based on thorough evaluation of information to decide upon its trustworthiness and its merit.

This definition suggests several key features of critical thinking that should be kept in mind as teachers and teacher educators. These include:

i. problem identification;
ii. the gathering of sufficient data to form an opinion on a topic;
iii. creative questioning;
iv. reasoned arguments;
v. active consideration of alternative explanations and opinions;
vi. evidence testing;
vii. thoughtful judgment;
viii. development of an independent opinion; and
ix. the sharing of results or opinions in a respectful manner (CTSM, 2013).

Figure 1.1 presents some of the most important features of critical thinking. These features demand different kinds of cognitive skills. Benjamin Bloom, a well-known educational psychologist, helped to develop a taxonomy, or classification scheme, of different domains of learning that included the affective (emotions and feelings), the psychomotor (physical skills) and the cognitive (intellectual skills). The cognitive domain is often the focus of educators; however, affective and psychomotor learning are also important for children and adults alike (Vanderbilt University Center for Teaching, 2016). In recent years, Bloom's original model (Table 1) of the cognitive domain has been expanded to include additional intellectual skills that develop as we move from lower-order to higher-order thinking skills (Table 2).

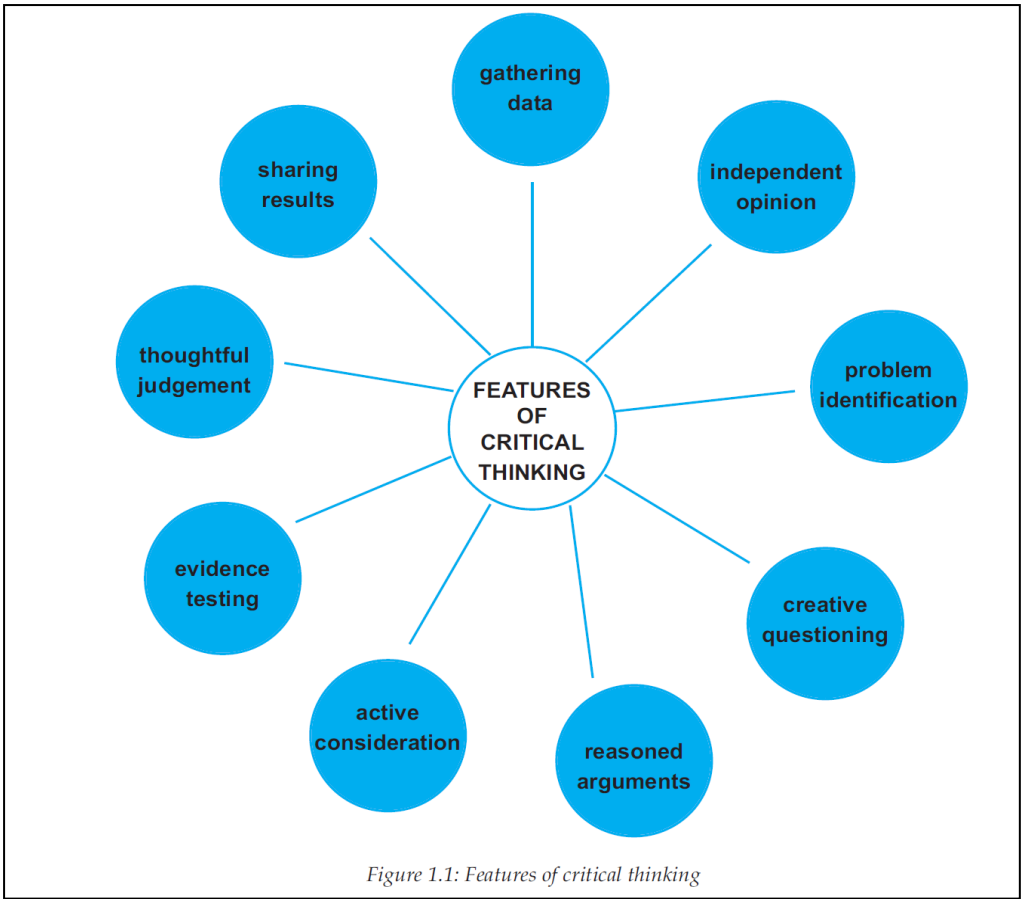

Figure 1.1: Features of critical thinking

Figure 1.1 Some of the most important features of critical thinking.

Table 1: Bloom's original taxonomy of cognitive domains

Lower order thinking skills ⟷ **Higher order thinking skills**					
Knowledge	**Comprehension**	**Application**	**Analysis**	**Synthesis**	**Evaluation**
define repeat list recall name relate underline	translate restate explain describe recognise express identify locate report review tell	interpret apply use demonstrate dramatise practice illustrate operate schedule shop sketch	distinguish analyse differentiate appraise calculate experiment test compare contrast criticise inspect question relate solve examine categorise	compose plan propose design formulate arrange assemble collect construct create set up organise manage prepare	judge appraise evaluate rate compare value score select assess estimate measure

Table 2: Bloom's revised taxonomy of cognitive processes: Categories, processes and alternative names

Lower order thinking skills → **Higher order thinking skills**					
Remember	**Understand**	**Apply**	**Analyse**	**Evaluate**	**Create**
recognizing • identifying **recalling** • retrieving	**interpreting** • clarifying • paraphrasing • representing • translating **exemplifying** • illustrating • instantiating **classifying** • categorizing • subsuming **summarizing** • abstracting • generalising **inferring** • concluding • extrapolating • interpolating • predicting **comparing** • contrasting • mapping • matching **explaining** • constructing models	**executing** • carrying out **implementing** • using	**differentiating** • discriminating • distinguishing • focusing • selecting **organizing** • finding coherence • integrating • outlining • parsing • structuring **attributing** • deconstructing	**checking** • coordinating • detecting • monitoring • testing **critiquing** • judging	**generating** • hypothesizing **planning** • designing **producing** • constructing

***Source**: Iowa State University Center for Excellence in Learning and Teaching (2012)*

It is important to note that not all of these levels of the cognitive processes promote critical thinking. For instance, the lower-order skills of remembering, understanding and applying are more basic forms of thinking, while the ability to analyse, evaluate and create are considered central to critical thinking. This sourcebook has a variety of methods and strategies for developing higher-order thinking skills that promote critical thinking.

Benefits of critical thinking

Benefits of critical thinking identified in the CTSM that affect the individual, family/workplace and society as a whole include the following abilities:

- to make complex decisions regarding what to do or believe
- to anticipate the consequences of one's decisions
- to settle disputes by using such attributes as being well-informed, trustful of reason, open-minded, flexible, fair-minded in evaluation, honest in facing personal biases and prudent in making judgments (CTSM, 2013; p. 6).

We can see how these abilities would lead to a fairer and more democratic society by virtue of people's ability to analyse problems, evaluate the consequences of their actions and create solutions that promote equity and justice.

Critical thinking and constructivism

In many countries, educational teaching methods are changing from the traditional teacher-centred approach where knowledge is transmitted to the learners who passively listen and acquire new knowledge, to a constructivist approach where knowledge is constructed or generated by learners. The constructivist approach to teaching and learning entails an active, mental process of development where the learners are actively involved in their learning. Constructivist teaching is based on the belief that learners are the makers of meaning and knowledge. Constructivist teaching fosters critical thinking and creates motivated and independent learners. Therefore, a constructivist teachersees critical thinking at the heart of the teaching and learning process.

A constructivist approach challenges teachers to create environments in which they and their learners are encouraged to think and explore. Young learners learn by resolving cognitive conflicts through experiences, reflection and metacognition (Seaver et al., 2000). For young learners, in particular, direct experiences are vital to the critical thinking process. Teachers present problems that are relevant to the learners and thereafter they give them time to discuss the problem, challenge each other's answers to questions and suggest possible solutions. Such an approach does not develop critical thinking in isolation but as an integral part of the curriculum.

Learning to think critically is a process that takes time and sufficient opportunities to do so. The constructivist teacher must give learners opportunities to explore ideas, to question and to take risks is to create a rich environment for critical thinking. It is evident when observing children that even those aged five to seven are capable of thinking critically. Not only do they talk about their thinking, but they also demonstrate emergent reasoning skills. By the time children reach the upper grades in primary school, they should be comfortable and capable in using cognitive skills from lower- to higher-order thinking.

Critical thinking and active learning

Constructivism is based on the premise that teaching and learning are active processes in which existing knowledge is analysed and evaluated, and new knowledge is created through interactions between teacher and learners in the classroom. Thus, we can think of active learning as informed by constructivism and as central to the critical thinking process. According to the CTSM, active learning "encourages learners to make sense of information by engaging in the learning process through participation in a structured learning activity to obtain desired learning outcomes" (CTSM, 2013; p. 35). One of the key words in this definition is *participation.* Participation can also include active methods that simultaneously promote psychomotor development, such as role playing and experiments. There are many ways to promote active learning, and the chapters in this sourcebook will provide suitable examples for specific subject areas.

The principles of active learning are closely linked with the broader principles of critical thinking. Five of the most important linkages are:

i. Meaning-making occurs when learners link their existing knowledge and new knowledge.
ii. Learning by doing is more powerful than only learning facts.
iii. Learning includes transferring knowledge from one domain to another.

iv. Learning involves the co-construction of meaning through interactions with others.
v. Speaking and writing are important ways for learners to articulate their understanding. (CTSM, 2013; p. 36)

These linkages apply to all learning areas or subjects, and we demonstrate how to make these linkages visible in Chapter 5, where we suggest ways to promote critical thinking across the curriculum.

Conclusion

This chapter has laid out the rationale for its use in Malawian schools. It has also introduced the concept of critical thinking. It has gone further to explain the relationship of critical thinking to cognitive domains and higher-order thinking skills, constructivism and active learning. The next chapter will discuss in greater detail on how to teach for critical thinking and characteristics of critical thinkers in the subject of science.

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CHAPTER TWO

Becoming a Critical Thinker in the Teaching of Mathematics

Introduction

Becoming a critical thinker is a conscious process that involves practice. One approach to promoting critical thinking is to incorporate it into the school curriculum. The final goalof this process is to become a person with reasonable judgment in problem solving and in life in general so that critical thinking becomes a routine. A critical thinker in mathematics asks what, why, when and how before acting or responding to a situation (or challenge), and checking what their actions imply for themselves and for others. Critical thinking in mathematics includes exploring or using algorithms but always along with assessing their significance. In this chapter, we explore characteristics of a well-cultivated criticalthinker in mathematics and the stages one passes through to become a cultivated critical thinker.

Characteristics of a cultivated critical thinker

Not everyone is a critical thinker, but we are all capable of becoming critical think if exposed to environments that promote critical thinking. To think critically is not always something that comes naturally, but it can be cultivated. In mathematics, critical thinking and problem solving are inseparable; they refer to the ability to use knowledge, facts and data effectively in order to arrive at solutions

The following are characteristics of a mathematical critical thinker:

(a) A critical thinker raises vital questions and problems by formulating them clearly and precisely. A critical thinker in mathematics asks questions to determine what a problem is all about and what it is that s/he is trying to figure out. A critical thinker identifies what it is that s/he wants to know, what is the given information to solve the problem, and what additional information is needed to solve the problem. A critical thinker also reflects on what similar problems s/he has solved before, what solutions could work, and work best, in this new situation, and how to test if a solution is reasonable.

(b) Critical thinking can be developed by thinking about and then attempting to solve real-life problems. Consider these two examples for a primary school learner:

Example 2.1

Frida and Jovin together weigh 63 kg. Jovin weighs 5 kg more than Frida. How much does Frida weigh?

Example 2.2

Mrs. Juma raises cows and chickens only. When she counts the heads of all animals, she finds 46 heads. When she counts the legs she gets 154. How many cows and how many chickens does she have?

A learner who thinks critically when trying to solve these problems will first read the questions and ask the following questions:

- What is supposed to be found?
- What information is given?
- What approaches should I take in order to find the answers?
- Is there a shorter and quicker way of finding the answers?
- Is the answer correct or not? If correct, is it unique?
- Could one solve the same problem using different numbers?
  For example, in 2.2, instead of cows and chickens,what if Mrs. Juma raised cows and goats? Would there be a solution?

(c) Gathers and assesses relevant information using abstract ideas to interpret problems (or situations) effectively. During this time, a critical thinker applies and adapts different techniques to effectively deal with problems and challenges. In particular, critical thinkers decide how to approach a problem for which they have no immediate answer or method; choose the most appropriate way to represent a mathematical situation; monitor their problem-solving progress and adjusts it as necessary; analyse their own response asking, "Does this make sense?"; communicate their mathematical ideas effectively to others; and connect their mathematical ideas, concepts and methods with their own lives and the wider world (The Critical Thinking Consortium, 2013). The following example illustrates how a critical thinker would gather and assess information to seek solutions to a given problem.

(d) **Example 2.3**

Peter's home is 750 m from school. He walks to and from school every day. After how many days does he cover 15 km distance?

A learner who is thinking critically, when working on this example will:

- *Choose the most appropriate way to approach the problem*: They may choose to work in metres or kilometres. They will need to know the conversion ratio, which is 1:1000, that is, 1 km is equivalent to 1000 m, and they need to understand how to use it. Therefore, 750 m is equivalent to $750 \div 1000 = 0.75 = \frac{3}{4}$ km. Using the same scale, 15 km is equivalent to $15 \times 1000 =$ 15,000 m. They will then choose to either work in metres or in kilometres.

If they choose to work in metres, they will find the number of days that Peter can travel 15,000 m. Common methods are:

Method 1	**Method 2**
After day 1 he covers $750 \times 2 = 1500$ m After day 2 he covers $1500 \times 2 = 3000$ m After day 4 he covers $3000 \times 2 = 6000$ m After day 8 he covers $6000 \times 2 = 12{,}000$ m After day 10 he covers $12{,}000 + 3000 = 15{,}000$ m.	Total distance ÷ daily distance $= 15000 \div 1500$ $= \frac{15000}{1500}$ $= 10$
Therefore, Peter covers 15 km after 10 days.	

If learners choose to work in kilometres, they will find 15 km as the number of days that Peter can travel. Common methods are:

Method 1	**Method 2**
After day 1 he covers $0.75 \times 2 = 1.5$ km After day 2 he covers $1.5 \times 2 = 3$ km After day 4 he covers $3 \times 2 = 6$ km After day 8 he covers $6 \times 2 = 12$ km After day 10 he covers $12 + 3 = 15$ km.	Total distance ÷ daily distance $= 15 \div 1.5$ $= \frac{15}{1.5}$ $= 10$
Therefore, Peter covers 15 km after 10 days.	

- *Monitor their problem solving progress and adjust as necessary*: In method 1 above, instead of continuing with days sequentially, critical thinkers will adjust by doubling the days to make their calculations quicker. Also, for the same purpose of speeding up the calculations, they add the distance travelled after 8 days to the distance travelled after 2 days to get to the final answer.

- *Communicate their mathematical ideas effectively*: In method 2 above, before dividing numbers, a critical thinker will state clearly what is being divided and will give the final answer in the appropriate units.

- *Connect mathematics to their everyday lives:* After completing the problem, learners may think of their own journeys to and from school and think of how many kilometers they cover in a week or so. At this point they will realize that the mathematics they learn at school is, in fact, applicable to everyday life, and they can communicate mathematical problems in everyday language.

(e) A critical thinker arrives at well-reasoned conclusions and solutions. In Example 2.3, if a learner gets 100 days, for example, as the final answer and leaves it there, they will not be thinking critically because the number (100) is large. Thinking in metres for example, Peter covers 1500 (a thousand and a half) metres a day. To cover 15,000 metres, Peter will need far less than 100 days. Another example where critical thinking helps to detect incorrect conclusions and solutions is the following:

Example 2.4

A mother shared K6000 with her three children,Chisomo, Mwandida and Jane in the ratio 1:2:3, respectively. How much did Jane get?

From the given ratio, and the reasoning outlined in Example 2.3 for a critical thinker, a learner who thinks critically and has learnt the content on ratios will come up with the correct answer of K3000. However, a learner who contends that Jane got K1500, will not be thinking critically because from the ratios given, for every Kwacha that Chisomo gets, Mwandida gets twice as many and Jane gets three times the amount. Jane will get the largest amount of the three. Considering this, the answer K1500 for Jane is clearly impossible. A critical thinker will not give an answer like this; they will reason first and realize the answer is impossible.

(f) A critical thinker thinks beyond their norm. They think open-mindedly within alternative systems of thought, recognizing and assessing their assumptions, the implications of these assumptions and their practical consequences. In addition, they are able to make connections between abstract ideas learnt in class and strategies to solve problems that do not have direct/explicit connections to everyday life. Consider the following example for primary learners from Mathematical Challenges for Able Learners (Department for Education and Employment, 2000).

Example 2.5

Albert Square

36 people live in the eight houses in Albert Square.

Each house has a different number of people living in it.

Each line of three houses has 15 people living in it.

How many people live in each house?

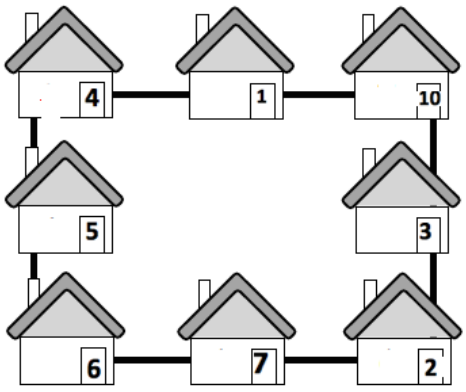
To answer this question, learners will need to think differently from when they are answering content-based mathematics questions and problems because there is no rule or formula directly associated with solving this problem. Getting the correct answers to this problem will require learners to engage in critical thinking by analysing mathematically the information given, e.g. what do the sentences "each house has a different number of people living in it" and "each line of three houses has 15 people living in it" imply? Then learners need to come up with an approach to get the correct answers. Being a critical thinker in mathematics is about being creative, innovative and inventive. In this case, learners would need to:

- Understand what information is being asked for. This is the number of people living in each of the eight houses in Albert Square.
- Collect all the information provided and see how they can be applied to solve the problem. It is given that there are eight houses arranged in rows and columns.

Each house has a different number of people living in it, and each row or column of three houses has a total of 15 people. These imply that:

- Each house has a person living in it
- Possible numbers of people for each house are 1, 2, …., 12. Cannot have more than 12 people in a house, to fulfill the two conditions given.
- Each house on the four corners cannot have more than 10 people otherwise houses will have the same number of people to fulfill the condition of 15 totals.

- Find strategies to solve the problem. One strategy is to start where you have fewer choices. So fill a house in a corner with the maximum number of people, 10. Then complete the corresponding row and column using the only two possibilities: 10+1+4=15, and 10+2+3=15. Next fill in the house at the corner diagonally opposite to where you started, as this will leave you with fewer choices for the remaining houses. Then complete the task making sure that the two conditions in Example 2.5 are satisfied. One answer is:



- Check your solution. Is this the only solution? Can you find others? Could there be a quicker way of obtaining solutions to this problem? This is where creativity, innovation and inventiveness will be demonstrated.

(g) A critical thinker communicates effectively with others to find solutions to complex problems, and this is also the case in mathematics. Learners should be able to distinguish inaccurate from accurate information and understand that people may lack relevant knowledge and may not always convey what they know. They should be able to ask questions clearly to help them get more information when it is needed and to get the most accurate information possible. They should also accept criticism when others find shortcomings in their work and value reasonable points from others.

Specific characteristics of critical thinking mathematics teachers

Learners in mathematics can learn to think critically only if their teachers are critical thinkers. Mathematics teachers who think critically have some of the following characteristics:

(a) They utilize a variety of questions, mainly open-ended questions that can be answered using multiple approaches, with the aim of encouraging learners to think

before responding. They use higher-order questions based on Bloom's taxonomy so as to promote analysis, evaluation, synthesis, and creating new knowledge. Effective, higher-order questioning builds learners' critical thinking skills and hence promotes independence.

(b) They know the subject matter to the level above the level being taught (Ball &Forzani, 2011; Kessel, 2009). Knowing the content well has a significant effect on what a learner can learn and hence on learners' thinking. For example, knowing the content well can help teachers to choose the most suitable questions to ask, the best examples and resources to use in class that promote higher-order mathematical thinking. Content knowledge also helps in understanding and detecting errors and misconceptions in learners' work, allowing teachers to provide prompt feedback. Willingham (2007) asserts: "The ability to think critically depends on having adequate content knowledge; you can't think critically about topics you know little about or solve problems that you don't know well enough to recognize and execute the type of solutions they call for" (p. 12).

(c) They create active learning environments that help learners to develop a repertoire of thinking tools that they can use confidently and independently. These learning environments provide opportunities for learners to explore their mathematical ideas and demands that teachers and fellow learners respect each other's opinions.

(d) They facilitate learners' ability to reach well-reasoned conclusions and solutions, and to express their views in spoken and written forms. Mathematics teachers who think critically help learners to appreciate that there are multiple ways of approaching mathematical problems, and they encourage learners to discuss approaches they find most accessible, repeatable and efficient. These teachers also allow the use of varied presentations of solutions as a means of encouraging different ways of communicating with others.

(e) They create lessons and activities for use in the classroom or other settings for the teaching of mathematics to encourage learners to reason within alternative systems of thought. This is important because it helps learners to recognize and assess their assumptions and the implications and practical consequences of their work. Critical thinking teachers also provide clear, explicit and timely feedback to learners, which aims at improving learners' learning rather than only evaluating their work.

(f) They guide learners to communicate effectively with others in their spoken and written work. Critical thinking teachers guide their learners in how to interact with others in their family and in their community to find solutions to complex problems.

Moving learners toward critical thinking when teaching mathematics

Critical thinking can be incorporated directly in the teaching of mathematics because it goes parallel with mathematical reasoning. To cultivate critical thinkers in mathematics, teachers should consider the following six practices:

1. Determine and share with learners the learning objectives for each of the critical skills in mathematics based on Bloom's taxonomy, including problem solving, application, analysis, synthesis, and evaluation. Identify key competencies to be learnt and write them in a language that learners will understand. Use action verbs

to list specific learning outcomes, for example: *Learners should be able to* ***calculate*** *the size of an interior (or exterior) angle of a regular pentagon;* ***construct*** *a regular pentagon;* ***justify*** *each step in their calculations that involve angles in a regular pentagon; etc*. Additionally, teachers should consider constructing success criteria with learners so they can own and assess their own learning; setting clear expectations for respectful participation in pair/group work; and letting learners ask for clarification when they disagree with others.

2. Model the way you, as a teacher, think when solving problems to help learners strengthen and practice their ability to articulate their thinking process. This can be part of scaffolding strategies that guide learners through the problem-solving process. Modeling thinking can also encourage learners to persevere during problem solving. It may also be helpful for teachers to model mathematical thinking from the learners' perspectives – as if the teacher is looking at the problems for the first time. For example, consider solving the following problem in class:

Example 2.6
In a village near Blantyre there are 650 families. Each day, every family receives a plastic bag from the village store for shopping. If all families in the village decide to use their own bags instead, how many plastic bags would be saved in a year?
To solve this problem, a teacher could model his or her thinking process by writing down information on:

- *What we need to find*: the number of plastics bags to be saved in the village per year if all families use their own bags instead of the bags given from the local village. The units for the final answer is "*bags*".

- *What information is given*: there are 650 families, so one bag will be saved per family per day.

- *What else we need to know*: the number of days per year, which is 365 days.

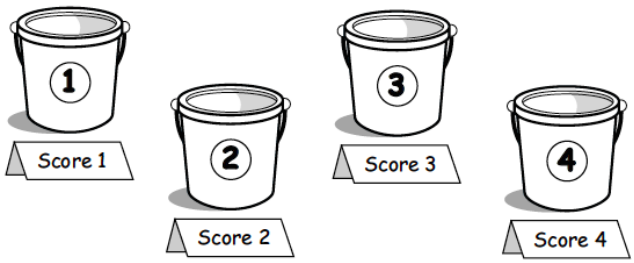
- *Which method will work best to solve the problem*: 650 bags are saved per day. In a year, we will use multiplication, 650×365.

- *Check the answer:* the answer must be greater than $600 \times 300 = 180{,}000$ but less than $700 \times 400 = 240{,}000$. It is closer to 240,000 than it is to 180,000 because 365 is closer to 400 than it is to 300.

This is an example of modeling in the classroom to help learners think critically when confronted by problem solving tasks in school and in daily life. Another important aspect of modeling is to teach multiple approaches to problem solving. For instance, in Example 2.4, the teacher can discuss easier ways of multiplying the numbers. When learners are given choices for methods, they get the opportunity to think critically by choosing methods that lead them to correct solutions quicker. By doing so, teachers are encouraging learners to be creative. For example, after solving Example 2.4 together in class, the teacher could ask learners to create similar word problems and give others to solve.

3. Teach through questioning to develop the critical thinking skills and include convergent (Bloom's lower-order thinking) and divergent (Bloom's higher-order thinking) questions. Divergent questions are open-ended questions and therefore have multiple approaches and, sometimes, multiple answers. They help learners to choose the best techniques in solving problems since there are alternatives, and hence they promote critical thinking. Teachers should choose questions carefully before each lesson, matching them with the learning outcomes and encouraging mathematics 'talk' in class. By using questions effectively, teachers are encouraging learners to discuss emerging points from instructions and/or problem-solving tasks, and they can take advantage of unexpected (or incorrect) answers to explore further new ideas and concepts. Questions that accommodate multiple approaches such as Example 2.5 or examples with multiple solutions like the problem below which is meant for Standard 1 learners:

   Example 2.7
   John threw three dice into buckets. Each die went in a bucket. The number on the bucket represents the score obtained when a die goes in. More than one die can go in a bucket.



   How many ways are there for John to score 9? List them. What are the other possible scores? (Department for Education and Employment, 2000)

4. Review, refine and improve lessons by monitoring learner engagement in activities and exercises and collecting feedback from learners. One way to do this is to let learners defend their solutions – why do they think they have arrived at the correct answer? Why do they believe their method is the most efficient of all the methods that have been learned or discovered? In the case of multiple solutions, let them choose solution which might be the best and provide time for learners to discuss and solve problems in pairs or groups. Critical thinking teachers should strive to create an active learning environment with very high expectations and provide room for learners to reflect on their actions, through involvement in learning activities. For example, they can model and encourage learners to ask themselves questions such as, "Where will I use this kind of mathematics in my life?"

5. Provide prompt feedback to learners that is both motivational and descriptive rather than solely evaluative. It is important for teachers to let learners know where they have done well and where they need to either improve or move to higher-level thinking, and how to do so. For example, a comment such as the following provides feedback on what the learner has done well and how the learner can move to a more advanced level: "Well done. You have constructed a pentagon starting with three intersecting circles, now connect the dots on your diagram and propose the name of the new shape." Teachers should also provide assessment frequently to improve

learners' mathematical skills and create opportunities for self-assessment. Teachers also need to make sure that there is a connection between learning outcomes, success criteria, teaching methods and assessment. Creating rubrics to help learners focus on the most important parts of a lesson or unit and making assessment practice transparent can help learners set their own learning goals and can support them in reaching their target.

6. Reflect after each lesson and, if possible, create a reflective journal to document events happening in the classroom and actions that might be taken to improve both teaching and learning. This journal can be very helpful for future reference when teaching the same topic, and it can help to improve current learners' critical thinking in mathematics by modifying the next lesson to encourage higher-order thinking. (Duron, Limbach, & Waugh, 2006*)*.

Conclusion

In this chapter, we have seen that there is a difference between knowing how to do mathematics (computations using formulas and learning concepts) and critical thinking in mathematics. Critical thinking in mathematics includes knowing how and why formulas work, as well as being able to use them. Both skills are taught by teachers who encourage critical thinking, but the latter needs particular attention as explained above. Every critical thinker in a mathematics class knows how to approach problem-solving tasks, but not everyone who knows how to solve problems in a mathematics class is a critical thinker. Teaching mathematics may be considered by some people to be an 'everyday job' that can be performed by anyone, even without preparation because they believe learners are learning it only to pass their exams. In fact, mathematics is a complex field of study, and cultivating critical thinkers in a mathematics class requires that teachers approach each day differently. Careful planning, execution and reflection are required in order to accomplish the mission of preparing critical thinkers in the mathematics classroom.

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CHAPTER THREE

Specific Critical Thinking Methods and Strategies for the Teaching of Mathematics

Introduction

Effective teaching of mathematics involves critical thinking. Every learner in a mathematics lesson that is being run effectively engages in critical thinking and therefore has a chance to become a critical thinker. Every mathematics lesson, at any level and in any community, aims at being effective, but many do not succeed. It is evident from research that the process of helping learners to develop critical thinking skills is not easy and takes time (see for example, Marcut, 2005). The process needs both teachers' and learners' determination as well as a well-planned lesson. This chapter will discuss critical thinking methods and strategies for the teaching of mathematics.

Factors to consider when using critical thinking methods and strategies

General considerations

There are a number of general considerations for mathematics teachers who seek to promote critical thinking with some of the most important discussed in this section:

(a) Use meaningful learning objectives and share them with learners. When learners understand the objectives, it helps them to focus their thinking. Use action verbs to define learning outcomes. For example, learners in Standards7 and 8, having learnt how to present data using pie charts during the lesson on understanding mode of a set of data, the general objective using an action verb might be: to *find* the most common class shoe size. Specific objectives (or learning outcomes) can be written in a similar manner: to *collect* class shoe sizes; *record* data on a table; *deduce* the mode by reading the table; *draw* a pie chart representing class shoe sizes; *tell* the most common class shoe size from the pie chart; and *describe* the value(s) obtained using the table and pie chart.

(b) Match teaching methods and strategies with the characteristics of learners, e.g., slowlearners, able learners or mixed ability. For example, the main activity for the objective mentioned above would be suitable for individual, pair, or group work depending on how it is planned. In the case of groups, teachers can use mixed ability groups with those thought to be slower given active roles of collecting shoe sizes from fellow learners, and those who are faster in mathematics can lead in the construction of atable and charts. It is a good idea to plan in advance extension activities for any group that completes their tasks before the planned time. In such cases, teachers might ask them to find other examples where the concept of mode may be applied to find averages.

(c) Adapt learning activities and materials to accommodate those who have learning difficulties. One way to do this is to draw on learners' experiences when introducing a concept, especially the experiences of those learners with less content knowledge.

For the lesson on mode, the teacher might start by asking learners the number of siblings they have and using the data to introduce the mode concept.

(d) Match class activities with particular school and community norms so that the activity does not violate rules about appropriate questioning. For example, in the above lesson, asking each learner to say what their shoe sizes is more appropriate than measuring learners' feet, as this may be regarded in some communities as invading learners' privacy.

(e) Match learning activities with the space and size of the class. If a teacher has a large, class, one might think of having some activities outside the classroom, depending on the school settings. Data collection activities often require space as learners need to move around speaking to others. Teachers should make sure they have enough space before the start of any activity and enough supplies, such as rulers or compasses for each learner or small group of learners to use them.

(f) Show that knowledge is not static. Let learners explore other methods of obtaining solutions from their current problems and decide which one is best based on evidence obtained. For example, in the data collection problem, learners can defend the use of mode to determine the most common class shoe size, compared with the use of other measures of central tendencies, such as median and mean. However, future classes could utilize activities in which median or mean would be the most appropriate measures to show learners that the best method varies depending on the information one seeks. (CTSM, 2013, p. 13).

Specific considerations for mathematics classrooms

There are also specific considerations that mathematics teachers should keep in mind as they prepare lessons designed to teach content and critical thinking:

(a) Consider the practical insights and real-world utility for mathematics use that learners can take away from each lesson. Relate each lesson to real-life applications, even in simple problem-solving procedures where one asks these questions: What is the problem? What information do I have about this problem? Which strategies/approaches will I use? Which one is faster and cheaper? Have I solved the problem? Could there be another solution? These questions will help learners to approach practical, real -life problems with care and thoughtfulness – and in so doing they will apply critical thinking to these situations.

(b) Pay particular attention to the level of mathematics proficiency, age, and gender of learners in choosing methods that will encourage and not inhibit their active classroom participation. Ask question that encourage the participation of every learner in class, give thinking time and avoid too many chorus (whole class) answers. Use gender-responsive pedagogy, which means choosing activities and examples to use in class that promote critical thinking for both boys and girls and that avoid stereotypical examples that imply mathematics, science and engineering jobs are for boys/men only.

(c) Think creatively about how to use locally-available resources to create teaching and learning recourses. A good example is the creation and use of number (and algebra)

cards in mathematics. Cards are frequently used and can easily be made from deserted boxes. Tops from plastics bottles are very useful as counters. Experience has shown that many learners believe the only job for people doing mathematics in colleges/universities is teaching. Thus, for those learners who do not like teaching, they might not see the need to engage in a mathematics lesson because they believe it is not for them. In this case, important local experts (accountant, engineer, doctor) can come to the classroom as resource persons to link mathematics taught in school to their professions and to the community service as a whole.

(d) Share ideas with colleagues in other subjects about how they use critical thinking methods in overcrowded and under-resourced classrooms and try these out yourself if this is your situation. One strategy is to pose challenging open-ended questions that lead to discussions. Share your strategies with your colleagues afterwards.

(e) Find ways to show learners that mathematics does not operate in a vacuum by bringing in examples where mathematics is applied locally and internationally. Here are some examples to show that mathematics can be applied in many contexts, locally and internationally.

Example 3.1
Ben wants to buy 6 pineapples for his class. He visits a local grocery shop and reads the following notice:

One pineapple costs K640.
Buy one pineapple get the second $\frac{3}{4}$ of the price, and the third $\frac{5}{8}$ of the price. To buy 4 or more pineapples, the price is $\frac{4}{5}$ of the original price, for each pineapple.

What is the best deal for Ben? How much will he pay if he takes the offer?

Example 3.2
The following are the current times in the respective cities shown below: Tokyo, 5:38 am (Wednesday); New York, 4:38 pm (Tuesday); London, 9:38 pm (Tuesday); and Moscow, 11:38 pm (Tuesday).

Plane 1 will leave Tokyo at 5:40 am (Wednesday) to fly to New York. The flight time for plane 1 is 12 hours and 50 minutes. Plane 2 leaves Moscow at 11:40 pm (Tuesday) on its way to London and then to New York. The flight time from Moscow to London is 3 hours and 50 minutes. Plane 2 stops in London for 30 minutes before

leaving for New York. From London to New York is 7 hours 10 minutes. Which plane will arrive in New York first and when will this be? Show your method and justify each step in the method.

These examples illustrate some of the specific considerations for mathematics teachers who seek to promote critical thinking. The following are even more detailed examples of methods that may be useful in teaching mathematics. These are examples and not an exhaustive list of possibilities for how to use critical thinking strategies in the classroom.

Exposition methods

Exposition methods aim at introducing new learning. Some of these are structured overview, brainstorming, Know-Want to Know, Learn (KWL) and INSERT (CTSM, 2013). In mathematics new learning is developed easily using learners' prior knowledge and experience. A mathematics lesson starts with the goals of the lesson so that learners understand what they will learn. Connecting what learners know and what they can do to new ideas and concepts is emphasised by many studies including Antony and Walshaw (2009); Askew, Brown, Rhodes, William and Johnson, (1997); and Sullivan (2011). In particular, using learners' experiences to introduce new ideas and concepts has proven to work for lower achievers (Sullivan, 2011). Moreover, critical thinking in mathematics aims at preparing young people to be productive members of their communities, and therefore instructional methods should focus on the following (Sullivan, 2011):

(a) ***Conceptual understanding***. This means that we teach mathematics through high-level presentation of materials to facilitate deeper understanding of mathematical concepts, operations and relations, and that we teach relationally rather than instrumentally. Instrumental teaching involves showing **how** to apply rules, algorithms and formulas, whereas teaching relationally includes showing **why** these rules work, as well as how they work. It is important to let learners understand how and why formulas work, as well as how to apply them. For example, when teaching division of fractions, many teachers provide the following formula without further explanation:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}, \qquad b \neq 0, c \neq 0, d \neq 0.$$

Instead, they should demonstrate how the formula works by explaining to learners using examples that this is a shorter and, most of the time, a quicker way of dividing fractions. In some examples, e.g., where fractions have a common denominator, this method takes longer, and learners can be guided to understand this difference as in the example below:

Example 3.3

Evaluate(a) $\frac{1}{7} \div \frac{4}{7}$(b) $\frac{3}{4} \div \frac{2}{5}$

Solution

Method 1 (without the formula)	Method 2 (using the formula)
(a) $\frac{1}{7} \div \frac{4}{7} = \frac{1 \div 4}{7 \div 7} = \frac{1 \div 4}{1} = \frac{1}{4}$	$\frac{1}{7} \div \frac{4}{7} = \frac{1}{7} \times \frac{7}{4} = \frac{1 \times 7}{7 \times 4} = \frac{7}{28} = \frac{1}{4}$
(b) $\frac{3}{4} \div \frac{2}{5} = \frac{3 \div 2}{4 \div 5} = \frac{3/2}{4/5} = \frac{\frac{3}{2} \times 2}{\frac{4}{5} \times 2} = \frac{3}{8/5}$ $= \frac{3 \times 5}{\frac{8}{5} \times 5} = \frac{15}{8}$	$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$

In part (a), the second and third stages of the calculations in Method 1 may easily be skipped, whereas the second and third stages for Method 2 are difficult to skip. This makes method 1 quicker compared to method 2, for part (a).

Proposed activity:

- Select one learner to state his or her understanding of a concept/rule or principle you introduced;
- Randomly select another learner to summarize what the first learner said; and
- Ask the first person if the second person accurately represented what she originally said. (Paul & Elder, 2014).

As well as enhancing learners' understanding of the rules and concepts, the activity helps them to stay engaged and actively participating in the lesson.

(b) ***Mathematical fluency***. This means teaching learners skills to carry out mathematics procedures flexibly, accurately, efficiently and appropriately. When giving tasks about problem solving, teachers should help learners to focus by giving clear instructions and clarifying key vocabularies so they can concentrate on the mathematics of the problem. It also helps to encourage learners to do some of the mathematical computations mentally to increase efficiency. Teachers can select appropriate instructional tasks and provide tools to complete those tasks efficiently and accurately like calculators, graph paper, compasses, etc., where applicable.

Proposed activity: 'Think of a number' activity to help learners develop algebraic skills, e.g.:

- Instructions that give the same final answer
- Instructions that result with the starting number

Teachers may formulate these instructions to suit the age and level of learners. They then ask learners to try these instructions with different starting numbers then compare their answers with friends and finally learners to explain their findings: Why do you end with the same number while starting numbers are different? Why end with the starting number? The activity may be conducted as whole class activity or group activity.

(c) ***Strategic competence*.** This has to do with formulation, representation and solving problems. Modeling the process of problem solving helps learners to develop skills to recognize, formulate, represent and solve problems. Demonstrating multiple approaches to problems and multiple presentations of solutions helps learners to develop flexibility and independence.

Proposed activity: The teacher poses a problem with multiple approaches and multiple solutions, for example the problem in Example 2.5. She then provides a thinking framework using self-questioning strategy to learners to follow when solving mathematical problems:

- What is the problem asking for? What do I have to do?
- What information do I need (or do I have)?
- What information don't I need (or don't I have)?
- What method can I use to solve the problem? Could there be an easier method?
- Is my solution correct? How do I check if my solution is correct?
- Have I completed everything?

(d) ***Adaptive reasoning*.** By this we mean logical thinking, reflection, explanation and justification to support mathematical reasoning. Teachers seek to help learners learn how to reason inductively. Inductive reasoning helps to draw conclusions from observations. Understanding inductive reasoning helps recognize when a conjecture (based on inductive reasoning) is true or false. The next example shows how inductive reasoning is used to solve a mathematical problem.

Example 3.4
Find the next two numbers in the following sequence: 0, 1, 1, 2, 3, 5, 8, … … … …

Solution
From the given sequence, first term=0, second term=1, third term =1, fourth term =2, etc.
It is clear that third term=first term + second term; fourth term = second term + third term; fifth term = third term + fourth term, etc. Therefore, any term is given by the sum of the previous two terms. From this we deduce that the next two terms in the given sequence are **13** and **21**.

Proposed activity: Fact or Opinion?
A fact is something that we can prove. Facts are true. An example of a fact is: Number seven is greater than number 5. An opinion is a belief, and it may or may not be true. Even when many people share the same opinion, it does not make it a fact. An example of an opinion is: Mathematics is the hardest school subject. In the blanks below, write an F if the statement is a fact and an O if it is an opinion. When you're done, discuss your answers with the class.

a) ---------- 13 is an unlucky number.
b) ---------- 2 and 5 are the only prime numbers that end with a 2 or a 5.
c) ---------- The perimeter of a square is 4 times the length of one side.
d) ---------- The product of any number with 1 is the number itself.

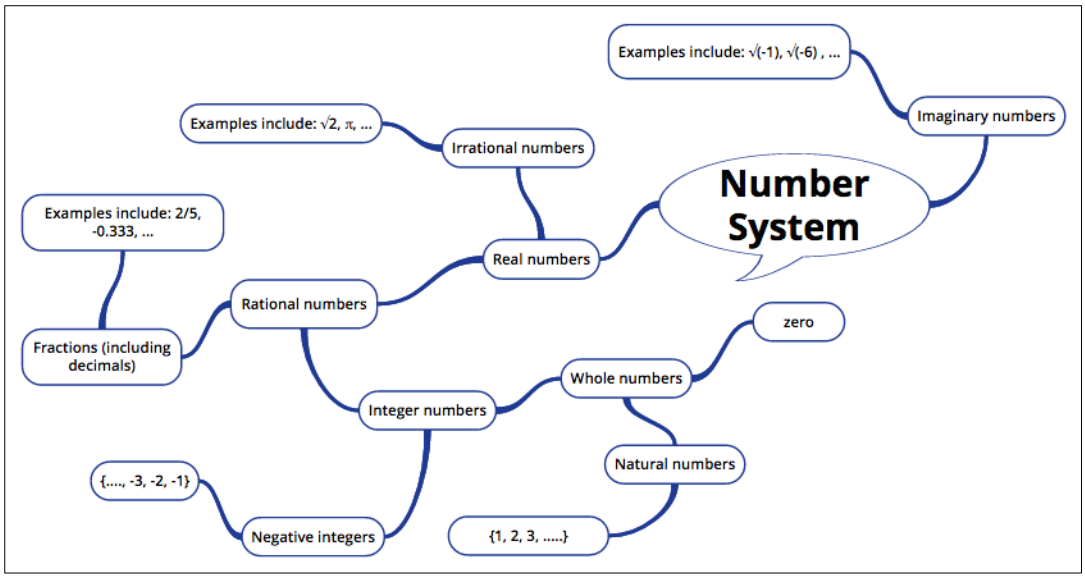
Example 3.5
Anne's birthday was on a Friday one year. She noticed that the next year it would fall on a Saturday, and it would fall on a Sunday in two years. Without checking, she said, "my birthday will be on a Friday again in seven years."

Inductive reasoning in mathematics helps learners to reason logically, including using appropriate counter examples where necessary in other practical situations outside mathematics. Teachers should help learners to use counter-examples to check whether or not their conclusion is generally true.

(e) Scaffolding. There are a number of scaffolding practices used in teaching mathematics, including modeling, collaborating, guiding, focusing, reflection and apprenticing. Scaffolding supports learners as they move from initial acquisition of a concept or skill to independent mastery. It therefore helps in bridging the gap between learning a skill and performing independently. Scaffolding is especially useful when inclusion is one of the teacher's goals. Every learner in a mathematics class should be given an opportunity to become a critical thinker. With scaffolding techniques, a teacher demonstrates a particular skill and learners watch and listen. They then perform the skill/concept independently under their teacher's guidance and support. For this reason, scaffolding strategies can be particularly helpful in creating an inclusive critical thinking classroom. According to the Researching Numeracy Project Team (2004), there are twelve scaffolding practices that may be used in a mathematics classroom to support learners' thinking. These are excavating, modelling, collaborating, guiding, convince me, noticing, focusing, probing, orienting, reflecting/reviewing, extending and apprenticing.

Co-operative learning methods
These are some of the most powerful and most common methods of teaching and learning mathematics, and they include Mix/Freeze/Pair (MFP), Paired Reading, Jigsaw, Walk Around-Talk Around, and one stay-three stray (CTSM, 2013). With all of these methods, learners in groups (assigned by their teachers or by themselves) work together to solve problems with the intention of learning from each other, thereby enhancing learners' social development. Learners typically enjoy working in groups with a variety of activities rather than sitting down and listening to the teacher for the entire lesson. Strategies that are part of co-operative learning are:

(a) *Brainstorming*. This can be used as a starter, during the main lesson or the concluding part of the lesson (the plenary). Using this method, learners can bring in new ideas about a concept, discuss the best strategy of solving a problem, or use it as a plenary and agree on the best solution to an open-ended problem. It also encourages critical thinking by showing that any idea is welcome, and it motivates learners to speak. One way of organising brainstorming ideas is mind mapping. Mind mapping is a way of writing down ideas and information in an organised way. It can be used to gather information on a concept or to solve a problem. The teacher starts with the main theme then breaks it into branches as desirable. An example of a mind map is the following:

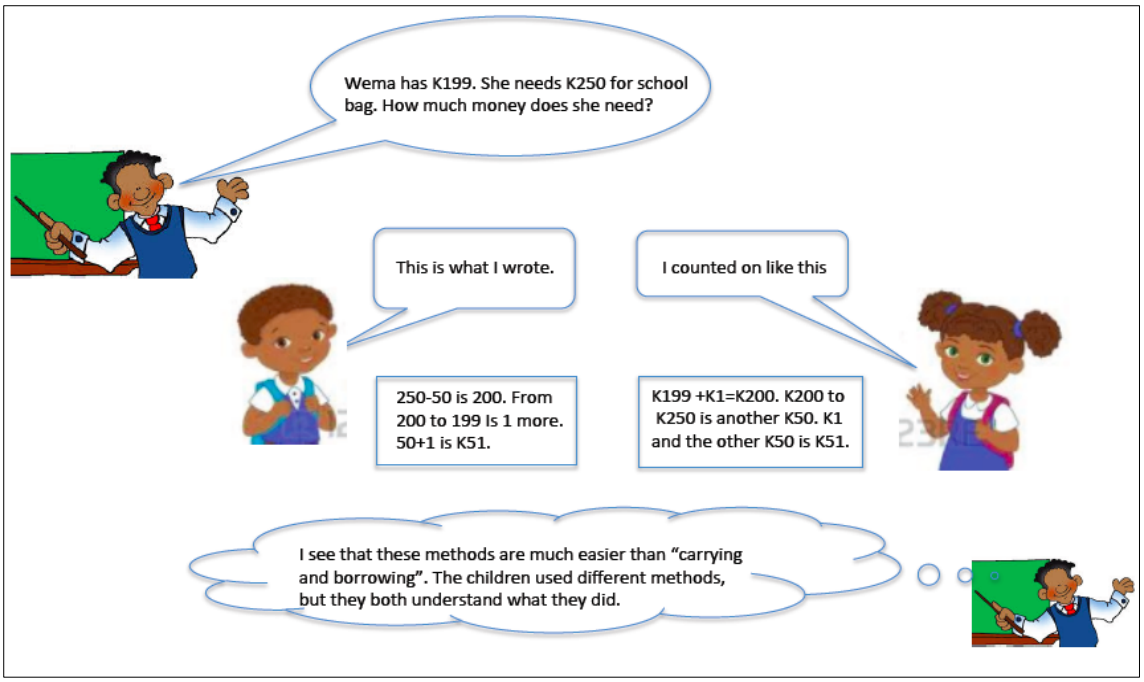


(b) *Differentiating challenges*. Putting learners in groups according to their abilities allows them to interact with others with similar abilities and can provide certain benefits for learners. The teacher can then support low achievers while challenging higher achievers in the same lesson. However, learner-centred teaching generally promotes mixed-ability groups to avoid stigmatization and to encourage learners to learn from those who are more advanced.

(c) *Mixed ability groups*. This is a common co-operative learning strategy by mixing learners with different abilities in mathematics in the same group. This is normally used during the main activities of the lesson. Learners work in mixed ability groups to complete assigned tasks or investigations and then write together reports, notes, or solutions. Teachers may then use these communications to summarize key mathematical ideas for the whole lesson or topic knowing that they reflect the general level of understanding of the entire class.

Nurturing engagement

This approach involves engaging learners in a mathematics class by making sure they stay on task and do so in an environment that nurtures their learning. Individually and in groups, learners are engaged using a variety of challenging tasks, questions, assignments, etc., to enhance their critical thinking capacity. Teachers should display high expectations to create a supportive atmosphere in class and to show learners that the teacher is interested in their thinking and that their ideas are valued. By doing this, teachers create a nurturing space and opportunities for learners to reason, communicate, reflect, share and critique their own practices. Here is what happened in one of the numeracy classes (COUNT, 2013):



In this classroom, the teacher encouraged learners to share their work with others and to foster relationship that can become a tool for developing learners' mathematical thinking, competencies and identities (Antony &Walshaw, 2009).

Mathematical investigation

This is an inquiry-based method of teaching and learning in which learners work through contextualized problem-solving tasks assigned by their teachers. Some of the activities that learners may engage in are: collecting information, recording information, presenting information, analysing information through answering and asking questions on a given task or topic. In this way, learners are encouraged to apply divergent thinking and come up with new ideas or proposition of multiple solutions (Diezmann, Watters, & English, 2002). Investigative methods in any subject help learners to investigate situations in their everyday life and come up with alternatives ways of dealing with situations, to raise doubts about answers, to test reliabilities and eventually to come up with reasonable decisions. An example of investigative task is working on a problem like the following:

Example 3.4

Is it true that the older you are, the taller you are? Use data from your immediate family to answer this question.

In this example learners will be asked to collect data individually from their own families and then combine their data with others to work in groups to complete the task.

Manipulative tools

These tools are often made from practical learning materials like sticks, counters, bead strings, digit cards, dice, dominoes, shapes and models, and are used in mathematics lessons. These hand-on tools are used not only in helping learners complete arithmetic computations, but also in making sense of abstract concepts in mathematics. For

example, using a model cube can help learners to differentiate vertices from edges and faces. Manipulative tools should be used to help learners in their mathematical thinking and reasoning. The use of manipulative is often justified by the ancient Chinese proverb:

> *I hear and I forget. I see and I remember. I do and I understand.*

However, current research indicates that using manipulative tools is not enough (see Marshall & Swan, 2008). Teachers should help learners to construct knowledge when using models and materials by connecting ideas through manipulating, thinking and talking. Swan and Sparrow (2004) suggest that for manipulative to be successful, the following line should be added to the above proverb:

> *I talk about it and I connect.*

Reflective methods

Learning mathematics through reflection is not a common practice, but, if performed effectively, it may help learners to think critically when dealing with challenging situations. Reflective practices in mathematics strengthen learners' mathematical thinking and, hence, their critical thinking. Having learners talk or write their daily experience in mathematics lesson(s) helps them to ask questions such as: What approach did I take? Why did I choose this approach? What was the question(s) asking for? What was my answer(s) compared to others? Could I have taken a different approach? Why did I respond to the teacher (or classmate) the way I did? Answering these questions help learners to think before responding the next time they come across a challenge or a task. Reflective questions could include, for example: What is the best thing I learnt today? Why do I think (or believe) that I learnt (concept name)? What did I find difficult in today's lesson and why? Why do I think (name of method) method of solving (name of problem) problem is the best method? Could I have changed my solution by removing the assumptions I made? Why did I impose these assumptions? An example of reflective writing from a 14-year-old learner in a mathematics class is:

> When I was in my last group, one of the girls got 10 and I strongly disagreed with her, because I thought 'if Julia started with 5', then she would have had just +3 to 5 which is 8, so how could she get 10? But when I came to this group, everyone got 8, so I thought I was right. But then I started to think about how Danielle would have got 10, so I started to work it out: 'Julia's last answer was incorrect, and she finished with 5' (that means she started with 7), but if she got it right, it would have been 7 + 3 and the answer would have been 10. Then I explained my answer to the rest of the table and they agreed. (Ryan & Williams, 2007, p. 49)

Conclusion

In this chapter, we have seen that the process of developing learners to be critical thinkers is long and requires thorough preparation, patience and resilience for both teachers and learners. The primary methods we reviewed in this chapter are exposition, cooperative learning, investigation, manipulative tools and reflection. These can be very useful in promoting critical thinking when teachers include them in a carefully-planned mathematics programme.

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CHAPTER FOUR

Mathematics Assessments that Stimulate Critical Thinking

Introduction

There are two types of assessment that are carried out in schools: assessment **for** learning and assessment **of** learning. The former is the type of assessment that happens during learning; it seeks and interprets evidence for use by a teacher and a learner to decide where learners are in their learning, where they aim to be, how to get there efficiently and what the next steps should be to advance their learning since learning is a continuous process. The latter type of assessment, known as summative assessment, is carried out most often at the end of a thematic unit, school term, or school year, and it's mainly done for the purposes of grading, ranking and reporting learners' performance.

Assessment for learning happens at every stage in the learning process and therefore informs teaching and learning in an ongoing manner. It is also known as formative assessment, and it goes hand in hand with critical thinking because the development of such thinking requires effective ways of assessing learning as it happens. This chapter discusses principles for developing assessment that stimulates critical thinking, both summative and formative.

General principles for developing assessment that stimulates critical thinking

The following are the general principles for developing assessments in mathematics that stimulate critical thinking.

(a) *Should reflect core educational values.* Learners leaving schools are expected to be numerate, in the sense that they are able to apply mathematical thinking skills effectively and confidently to their everyday lives. In this context, assessment in mathematics should consider how to assess the skills that are valuable for learners learning as well as the content itself. For instance,a teacher can include questions or problems with direct applications to daily life.

(b) *Most effective when assessment reflects an understanding of learning as multi-dimensional, integrated, and revealed in performance over time*. Both formative and summative assessment should reflect on what learners can do with what they know. In mathematics, problem solving methods, strategies and reasoning must be part of what is being assessed because this is where learners will demonstrate critical thinking and understanding of what they have learnt. They should be asked to show their methods and explaining their reasoning and choices.

(c) *Requires attention to outcomes and experiences that lead to those outcomes.* Assessments help to understand the output from a particular programme, say, those who completed one-school term, year or primary school leavers. The information may be needed for various reasons including to improve the programme itself – content, teaching and learning experiences. To get the desired

information assessment in mathematics should be differentiated; indicate clearly what kind of responses from learners will be regarded as higher order reasoning and vice versa. This will show which learners learned best and under what conditions.

(d) *Works best when it is ongoing not episodic*. To ensure improvement in teaching and learning programmes over a longer period of time, assessment should occur on a regular basis rather than happen only at the end of a term or school year. By using ongoing, formative assessment, teachers can modify their methods as needed, and learners can identify early on where they need more assistance to reach learning goals.

(e) *Has more of an impact when it begins with issues of use and illuminates questions that learners care about.*If learners cannot see the connection between an assessment and the issues they care about, the assessment process reduces itself to a routine where both teachers and learners can go through an assessment process but with little meaning for them. In contrast, when assessments are based on the curiosity of learners, they develop particular interest in the feedback, using it to evaluate their own learning and repositioning themselves for improved learning experiences (see CTSM, 2013).

(f) *Most likely to lead to improvement in class performance when it is part of a larger set of conditions that promote the teaching and learning processes.* Assessment should aim at enhancing the quality of teaching and learning in schools.In particular, formative assessment should inform the teaching and the learning processes with the intention of offering immediate feedback and greater improvement.

(g) *Assessment as part of being responsible to learners and to the public*. Assessments must be prepared in such a way that outcomes are used for the benefit of learners, parents and the public in general. After assessments have been completed, learnersshould be given information about their progress in terms of learning goals and expectations. Similar information may be given to parents so they can help support learners' learning. The public should also be made aware of what is going on in their schools and information from learners' performance is one of interest (CTSM, 2013). They, too, want to know about the quality of teaching and where schools in their community lie in the performance of mathematics in a district, region or country.

Specific principles for the assessment of mathematics that stimulate critical thinking

This section will discuss specific principles for summative and formative assessment in mathematics.

1. Summative assessments, although carried out periodically and not effective for providing information at the classroom level to make instructional adjustments and interventions, measure certain aspects of the learning process. These aspects include: learned skills and concepts; creative thinking; inquiry; evaluation; information processing; problem solving; reasoning; communication; etc. These are important aspects of critical thinking. The following are some of the assessment

criteria for summative assessment in mathematics that promote critical thinking. They are based on the general characteristics of a critical thinker described in Chapter 2.

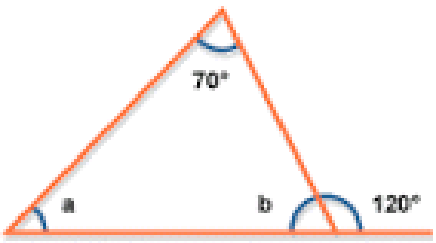
(a) *Raising vital questions*. This means that learners are able to formulate clearly and precisely vital aspects of the problem or reformulate the problem/question. For example,they should be able to assess how information given in a word problem is written using equations or expressions:

Example 4.1
The Game Shop charges a MWK1200 rental fee plus MWK900 per hour for renting a game. Peter paid MWK8400 to rent a game. How many hours did he stay with the game?

A learner who thinks critically will subtract rental fee from the total amount that Peter paid and then divide the answer by the hourly cost, giving the final answer in hours.

(b) *Gathering and assessing relevant information*. Critical thinkers should be able to show clearly the method(s) used to solve a problem or conduct an experiment. If it is a formula, then each item/term in the formula must be identified and if it's not the formula then each step justified. The following is an example:

Example 4.2
Find angles marked with the letters 'a' and 'b' in the diagram below. Give reasons for each answer.



A critical thinker will answer as follows:

b= $180 - 120 = 60^{\circ}$. Angles on a straight line add up to 180 degrees (reason).

a= $180 - (70 + 60) = 50^{\circ}$. Angles in a triangle add up to 180 degrees (reason).

A learner who gives answers without reasoning and/or without units will not be thinking critically because an answer of a=50 could imply anything. It is therefore is an incomplete answer.

(c) *Arriving at well-reasoned conclusion*. Learners should be able to demonstrate in any summative assessment how they arrived at their judgments or conclusions based on evidence. For example, look at the following problems:

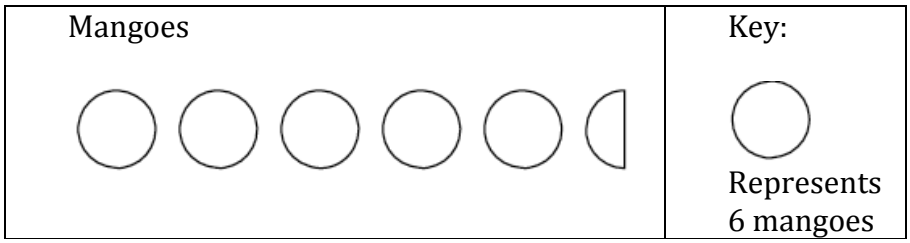
Example 4.3
Bibo says "*the sum of two prime numbers is always an even number*" Is Bibo correct? Explain.

A critical thinker at any level of schooling will start by thinking of any counter example to nullify the statement, if any. This is because it is easier to nullify than to prove. Since there is only one prime number that is different from others, not odd, one will start:

$2 + 3 = 5$, for example. When two prime numbers are added, the sum is odd. Therefore,Bibo is not correct. In fact, any other prime number added to 2 will serve as a counter example.

(d) *Integrate knowledge from different topics in mathematics*. Use different sources of knowledge to fully understand the question and give powerful interpretations and judgments. Examples of the questions that may be included in tests or exams to assess this criterion are as follows.

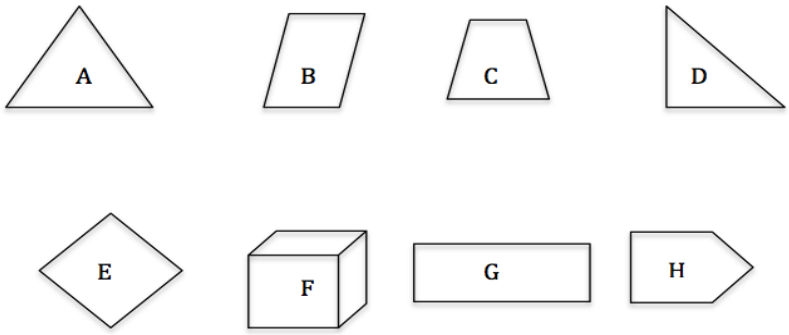
Example 4.4
The pictogram shows information about the number of mangoes that Mrs. Jackson has in her kitchen.



You need 8 mangoes to make 500 ml of mango juice. Mrs. Jackson wants to make 2 litres of mango juice for her family. Does Mrs. Jackson have enough mangoes? You must show all your working.

A learner who thinks critically will show
- Conversion of units, either 500 ml to 0.5 litres or 2 litres to 2000 ml to make sure that they work with same units.
- To make 2 litres of mango juice, Mrs. Jackson will need $8 \times 4 = 32$ mangoes, since 8 mangoes are need for half a litre.
- Interpretation of the key; that there are $5 \times 6 + 3 = 33$ mangoes in Mrs Jackson's kitchen.
- Therefore, Mrs. Jackson has enough mangoes.

Example 4.5
a) What properties do the shapes listed below have in common? Write a sentence about each set.



(i) B, E and G ……

(ii) A and D ……

(iii) C and H ……..

b) Pick the odd one out and give a reason for your choice.

For part (a), many learners will probably give answers like (i) they are quadrilaterals; (ii) they are triangles; (iii) they are polygons. But, those who will demonstrate critical thinking will read the question and realize that the question is asking about properties and therefore, for these, possible answers will be (i) have two pairs of parallel lines; (ii) have three angles; (iii) have **only** one pair of parallel lines.

For (b), while it is easy to pick the odd one out, giving a reason is not straightforward. Those who are not critical thinkers will probably come up with "it's a three-dimensional shape while other shapes are all two-dimensional". This is not enough to demonstrate critical thinking. It should be stated clearly that while other shapes are flat, with only two dimensions, shape F has an additional dimension; a depth. Therefore, shape F is an odd one out.

2. Assessment for learning or formative assessment is carried out frequently, in classrooms. For this to be effective in promoting critical thinking in a mathematics class, the following principles should be adhered to. With each one, examples of how to assess learners are provided (Assessment Reform Group, 2002; Flórez& Sammons, 2013).

(a) ***Should be part of effective planning of teaching and learning***. Specific objectives or expected learning outcomes should be explicit and transparent to the learners. Success criteria which are used by teachers and learners for assessment should also be challenging and of high quality. Both of these, objectives and success criteria, should be shared with learners at the beginning of the lesson so that learners can use the language in the objectives and success criteria in their discussions. Consider examples below. Teachers may need to remind learners of the learning intentions during learning. Whenever possible, teachers should involve learners in creating success criteria once the objectives have been understood. When learners understand learning intentions, they can

take control of the learning because they can determine whether they have successfully completed their work and where they have made unsuccessful attempts. In doing so, their critical thinking is enhanced because they are analysing and evaluating their own work. For example, when the general learning objective is to create and solve word problems using simple equations, success criteria may be:

- Create two word problems that may be solved by formulating corresponding simple equations.
- Use three different methods to solve a simple equation.

In these examples, during discussions learners will use sentences such as:

- I/we have created two word problems that I/we can solve by formulating simple equations from the problems. I/we have therefore met the first success criterion.
- I/we have used guessing, balancing, and trial and improvement methods to solve simple equations. I/we have therefore met the second success criterion.

(b) ***Should focus on how learners learn using effective questioning.*** As well as knowing what learners should learn, both teachers and learners must be aware of how learners are going to learn it. Chapter 3 discusses methods and strategies of teaching mathematics that promote critical thinking. When the teaching and learning processes are happening, one method that cuts across all methods is the questions and answers method. This method is effective in guiding learners towards the intended learning goals. It is therefore key to effective formative assessment. To promote critical thinking and encourage responses from learners, responses that involve mathematical reasoning, teachers should consider the following when asking questions:

Ask higher-order thinking questions. Refer to Bloom's Taxonomy (see Chapter 1 and Thompson, 2008) and create problems that call on learners to use these higher-order skills. For example, a question like "How would you *classify* the following shapes into triangles and quadrilaterals?" requires them to use classification—a higher-order cognitive skill—as they examine the properties of both triangles and quadrilaterals.

Give thinking time. Let learners think about possible solutions to any questions posed before asking them to respond. This will provide learners with opportunities to give meaningful answers (or sensible trials) and will encourage those who are slower to respond and share their thinking as well. When this happens, the teacher receives more feedback from the class as a whole to support and guide their learners' progress.

Involve as many learners as possible in answering and asking questions. This is important so that everyone feels they are respected and are part of the learning community. Teacher should frequently pick even those learners who are not putting their hands up because these learners may also know the

answer or have an attempt at an answer that would benefit the entire class if they heard it. The more learners are involved in their learning, the more they are prompted to think, and think critically, because they are more motivated and engaged.

(c) Provide feedback. Feedback is the most powerful tool that teachers have to help improve learners' learning, and it is a formative type of assessment if it is given frequently and in a timely manner. To give learners effective feedback that promotes critical thinking, consider doing the following in your mathematics class:

Engage learners in self-assessment and self-reflection. Let learners talk about (or display) their work and allow them to demonstrate their understanding of concepts or methods of problem solving in different ways.

Remind learners frequently of the learning outcomes and success criteria. This will guide them to where they are going in the lesson. Display high expectations for learners and describe to them the qualities of high-quality work in relation to the learning objectives and success criteria. This can be done by using models and examples of strong and weak work. For example, if the success criterion is "show everything you did to solve the problem," then skipping even one step is evidence of weak performance and therefore needs improvement.

Use descriptive (rather than evaluative) feedback. Provide clear and specific information in the form of oral or written comments to help learners understand what they need to do in order to improve. Involve learners in setting up goals to excel during feedback. Meeting the intended learning outcomes should not be the limit. Allow and support higher achievers to go beyond learning outcomes (or above success criteria).

Examples of descriptive feedback are:

- *This is quality work because you constructed two word problems, formulated corresponding simple equations and solved them using three different methods.*

- *Two things to improve: on constructing word problems, try and include negatives numbers and in solving simple equations, show all the steps when using trial and improvement method.*

- *You might try using words like deduct, difference, decrease, etc., in your word problems. So construct two more word problems using words similar to these. For the trial and improvement method, it helps if you state clearly, the answer is "too big" or "too small" until you reach the correct answer.*

(d) ***Encourage peer assessment, self-assessment and joint goal settings to cultivate self-monitoring and critical thinking learners***. When a teacher has

created an atmosphere of co-operative learning in the classroom, it is easier to encourage peer assessment (and self-assessment) because learners will feel more comfortable identifying the weaknesses as well as the strengths of their fellow learners' work and of their own work. Research indicates that peer assessment is often more easily comprehensible than assessment by teachers because of the use of similar language by peers (Flórez& Sammons, 2013; Thompson, 2008; Black &Wiliam, 2009). Below is an example of a peer evaluation form that could be tailored to a mathematics class: (source: http://www.lapresenter.com/coopevalpacket.pdf)

Quick Peer Evaluation Form

Name_____Class Period_____Date_____

Write the names of your group members in the numbered boxes. Then, assign yourself a value for each listed attribute. Finally, do the same for each of your group members and total all of the values.

Values: 5=Superior 4=Above Average 3=Average 2=Below Average 1=Weak

Attribute	Myself	1.	2.	3.	4.
Participated in group discussions.					
Helped keep the group on task.					
Contributed useful ideas.					
How much work was done.					
Quality of completed work					
Totals					

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Learners can use this form to assess themselves and other members in their working group. Another example of an assessment rubric is the one below for self-assessment on an independent task or at the end of the lesson (Learning Goals and Success Criteria, 2010):

Steps	Criteria	√
Understand the Problem	I re-read the question.	
	I underlined important words.	
	I put the question in my own words.	
	I identified the information that was given.	
	I identified what I need to find out.	
	I explained the problem to a partner.	
Make a Plan	I thought about how this problem is similar to problems I have solved before.	
	I thought about different strategies from our class strategy list.	
	I chose one or more strategies from the list.	
	I discussed my ideas about why the strategy would work with a partner.	
Carry Out the Plan	I tried the strategy until I solved the problem.	
	If necessary, I tried a different strategy or I asked for help.	
	I used words, pictures and/or symbols to show the steps I took to solve the problem.	
Look Back at the Solution	I checked to see if my answer makes sense.	
	I checked to see if there is a better way to solve this problem.	
	I wrote a short explanation describing how I solved the problem.	
Adapted from The Ontario Curriculum, Grades 1-8, Mathematics		

After completing the peer assessment or self-assessment rubrics, there might be a need to set up goal(s) for learners to work on in order to improve or to achieve higher grades. One way to do so is to use the SMART strategy for goal setting. SMART stands for Specific, Measurable, Achievable, Relevant and Timely (Blanchard, Zigarmi, &Zigarmi, 1985). To be specific, learners should say exactly what they will do; measurable means that learners say how they will know if they have met their goal; achievable requires that learners specify the steps they will take to reach their goal; relevant means that learners state what makes a goal important to them; and timely refers to when learners want to complete their goal. An example of a goal that may be set by a learner with the mathematics teacher's support is:

S I want to be able to construct two word problems whose simple equations include a negative number.

M I will construct corresponding equations showing negative numbers.

A I will practice the construction of word problems and corresponding equations using my notes and my textbook. I will do one problem per day after school and give it to my teacher to mark, the next day.

R I want to be a competent word problem solver.

T I will complete this task in a week – on 14th November 2016.

Conclusion

This chapter has focused on assessments that can be used in mathematics classes to promote

critical thinking. We have given particular attention to formative assessment because learners' difficulties in learning mathematics is often as a result of lack of ongoing feedback on how well they are reaching learning objectives with only final, end-of-term summative assessment being used in some classrooms. When learners understand that the purpose of assessment is to enhance their learning throughout a unit, they tend to concentrate better and participate more. In addition, the chapter has pointed out that low achievement is often the result of learners failing to understand what teachers expect them to do and how well-designed assessments can improve this situation (Black &Wiliam, 1998).

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CHAPTER FIVE

Integrating Critical Thinking Across the Curriculum

Introduction

The development of critical thinking is an important goal of primary education in Malawi. According to Beyer (1985), "critical thinking has two important dimensions. It is both a frame of mind and a number of specific mental operations" (p. 271).Having a critical spirit is as important as thinking critically. The critical spirit requires one to think critically about all aspects of life, to think critically about one's own thinking and to act on the basis of what one has considered when using critical thinking skills (Norris, 1985). Being a critical thinker also requires developing particular attitudes or dispositions such as respect, cooperation, ability to listen and to deliberate.

A very powerful environment for promoting critical thinking is created when teachers are able to integrate it across the curriculum by thinking about how to connect their particular subject-humanities, languages, mathematics or science—with other subjects. Such connections are useful for developing learners' higher-order cognitive skills, such as analysis, evaluation, and synthesis. When one subject area reinforces content in the others, learners begin to identify and understand relationships between concepts and develop new understandings across the curriculum.

This chapter will discuss ways for integrating critical thinking across the curriculum and provide examples of how teachers in humanities, languages, mathematics and science can support the learning of content and critical thinking skills in each of their subject areas.

School-based support for integration of critical thinking across the curriculum

It can be a challenging experience for teachers to design lessons that create opportunities for critical thinking *and* to find ways of supporting learning of other content areas. However, learners who experience such integration learn to synthesise information from multiple sources and evaluate it in a more sophisticated manner. For instance, learners may learn in their life skills class about how to prevent malaria, but their learning moves to a higher order when this information is connected to a language activity in which learners write about a time when they were sick and to mathematics lesson in which learners learn to multiply numbers using the reproduction of malarial parasites in the bloodstream as an example. When content and critical thinking skills are infused throughout the curriculum, there is continual reinforcement of core content throughout the school day and opportunity to think about it from different perspectives.

School leaders, such as head teachers, play a critical role in creating the conditions that allow teachers to integrate critical thinking across the curriculum. Teachers need to work together and determine the following:

- How best to prompt learners to use different strategies to think about the content they are learning in each of their classes.

- How to encourage learners' development of higher-order thinking skills that can be used in every subject area.
- How to relate the content in one class to the content of others to stimulate greater interest by learners in what they are learning.

The following strategies can be used to create a holistic, school-wide approach to teaching critical thinking:

- Teachers work with school heads to develop a system by which teachers who teach the same learners (all Standard 3 teachers, for example) are allocated time every two-three weeks to share with their fellow teachers the content (topics) they are working on in their humanities, languages, mathematics or science classes and how they are trying to promote critical thinking during these lessons.

- Teachers identify vocabulary, stories, experiments, field trips and other activities that would reinforce the learning of content across two or more subjects, and they share with one another the specific critical thinking skills they seek to develop in carrying out these activities.

- Teachers visit their fellow teachers' classes whenever this is feasible and provide peer feedback in a supportive, non-evaluative manner on how their colleagues could continue to improve their use of critical thinking methods and strategies to teach content across the curriculum.

- School heads allocate time each month when teachers can share with one another a particularly effective method for promoting critical thinking they have recently used in the classroom. They can also use this time to discuss how the school can promote curriculum integration through the development of themes that are relevant to more than one subject and that promote critical thinking. If time is limited, two teachers could be selected each month to share examples of how they are working to integrate a common theme in their classes and the critical thinking skills they are building into this theme. As teachers become more comfortable with this form of school-based professional development, they can also be encouraged to discuss efforts at integration that did not work as well as planned or critical thinking methods that did not seem effective. Their colleagues can provide feedback on how to make their efforts more successful.

Classroom-based strategies for critical thinking integration across the curriculum

Teachers can do a great deal to encourage critical thinking and curriculum integration. It can be done in many different ways using simple strategies, such as, on a Friday, asking different learners during the humanities class to mention something they learned in their language class that relates to their learning in humanities that week. This strategy builds oral proficiency and can be used to develop summarizing skills. Critical thinking and curriculum integration can also be much more elaborate, as when an accountant from the community is invited to come to class and talk about how s/he uses mathematics, and the language and literacy teacher, later the same day or the next

day, asks learners to role play being an accountant and client or to write a story about an accountant who does not know how to multiply numbers.

In preparing to use an integrated approach, teachers should:

- share their plans with their fellow teachers to make sure important content has been covered or activities (such as a guest speaker) have occurred as scheduled;
- identify the concepts and skills in their content areas that they seek to reinforce by drawing on content in other classes;
- asks reflective questions to guide learners toward analysis, evaluation and synthesis of concepts or facts learned in two or more subject areas; and
- reinforce critical thinking by providing ample opportunities for learners to engage in thinking independently.

In integrating critical thinking across the curriculum, it is important for teachers to consider the following points about learning:

- The interdependence between content and thinking means that teachers should strive to incorporate critical thinking skills into a wide variety of subjects, situations, contexts and educational levels. Learning a subject means learning the terminology, understanding the concepts, being able to discuss and write about it critically and to transfer this subject knowledge to other contexts.

- Critical thinking can and should contribute to achieving learning outcomes through assessment. This can include content understanding, recall and communication skills as well as skillfulness in thinking.

- Interaction during instruction should be encouraged because it allows learners to share ideas, confront different and even conflicting perspectives, challenge views of other people or to be challenged and defend their own perspectives. They learn from each other by collaborating and deliberating, all of which ultimately contributes to their critical thinking. Such interaction creates a constructive avenue for learners to learn and experience a democratic way of living.

- Classroom conditions that stimulate critical, creative thinking are necessary so that learners are encouraged to make inferences, think intuitively and spontaneously and use inquiry-discovery teaching techniques. Learners need to feel free to suggest and experiment with new ideas and approaches.

Specific strategies: Integrating humanities, languages, mathematics and science

(a) *Word walls*: Teachers can identify a space on one wall in the classroom where important vocabulary can be recorded. The word wall can be specific to one subject (Figures 5.1a and 5.1b) or, if teachers share a classroom, the word wall can have key terms from different subjects on it (Picture 5.2) This kind of visual reinforcement of key words helps to build language and literacy skills, and it encourages learners to synthesise vocabulary learned in different subject areas if teachers develop questions and other activities that require them to do so.

Figure 5.1a: Science word walls

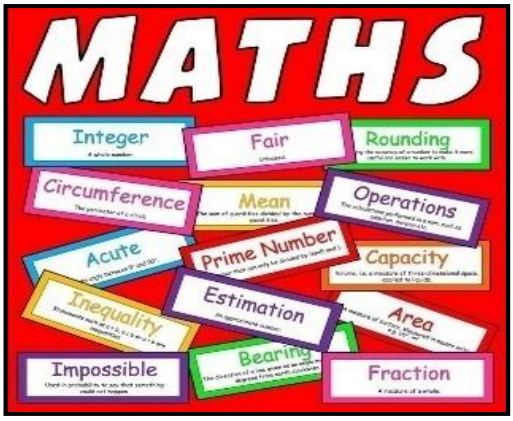

Figure 5.1b: Mathematics word wall

Figure 5.2: Multi-subject word wall

(b) *Word tree*: A word tree can also be used to stimulate lower order and higher order questioning in the humanities, languages, mathematics and science (Figure 5.3). This word tree can be exhibited on the classroom wall and a conversation or a questions session to stimulate critical thinking on any topic could be guided by these questions. This strategy is suitable for use in Kinder or lower primary and senior primary.

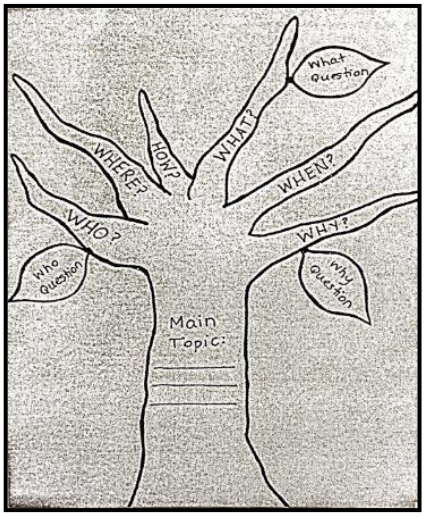

Figure 5.3: Multi-subject word tree

(c) *Drama:* There are many ways that drama in the form of role-playing and plays can be used to develop content knowledge in the humanities while also reinforcing listening, speaking, reading and writing skills in the language classroom. Religious education has stories that can be retold in the form of a full-length play or as a short role play, and the learners could be required to use certain word forms or grammatical structures that they are studying in their languages class. A play could also be developed that addresses the topic of gender and entrepreneurship from the life skills component of the humanities curriculum. In the play, learners could have some of the characters doing the mathematical calculations they have been studying in their mathematics class. By developing their own script, learners would have to synthesise information from life skills and mathematics classes and analyse the kind of mathematics problems an entrepreneur in a particular field might need to do.

(d) *Stories and story problems*: Every subject in thetheir critical thinking skills.

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